

MATHCOUNTS[®], Problem of the Week Archive

Happy Pi Day! – March 9, 2009

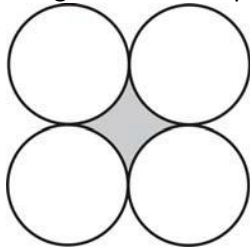
Problems & Solutions

March 14 is Pi Day. Pi represents the ratio of the circumference of a circle to its diameter. For the following problems use 3.14 as an approximation for π .

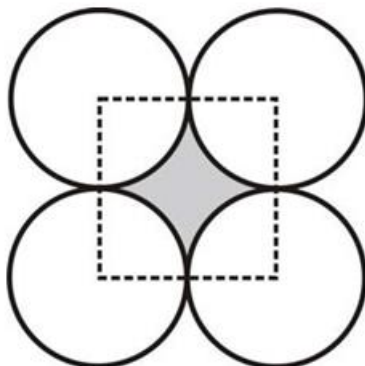
Charlize has a fence around her 15-foot by 20-foot garden. If she decides to reconfigure her garden into a circle and she only wants to use the fence she already has to enclose it, what is the area of the largest possible circular garden Charlize could create? Express your answer as a decimal to the nearest hundredth.

*First, we need to figure out how much fence she has. Her original garden is 15 by 20, so she must have $15 + 20 + 15 + 20 = 70$ feet of fence. The largest circular garden she can create will use all of the fence that surrounded the old garden (thus, the perimeter, or circumference, of the circle should be 70 feet). If the circumference is 70 feet, the diameter is $70/3.14 = 22.29299363$ feet. This means the radius is $22.29299363/2 = 11.14649682$ feet and the area is $(11.14649682)^2(3.14) = 390.127388865$, which is **390.13** to the nearest hundredth.*

The figure below is made up of four congruent circles. Each circle is tangent to two of the three other circles, as shown. If the radius of each of the circles is 6 units, what is the area of the shaded region in the figure below? Express your answer as a decimal to the nearest hundredth.

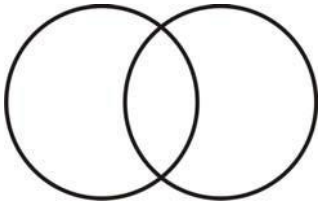


If we draw radii from the centers of each of the circles to the points where the circles are tangent, as shown below, we can see that a square is formed.



The sides of this square are $2(r) = 2(6) = 12$ units long, and thus, the area is $(12)(12) = 144$ square units. If we subtract the area of the four quarter-circles that are in the corners of the square, we are left with the area of the shaded region. So, $144 - 4(1/4)[6^2(3.14)] = 30.96$ square units, to the nearest hundredth.

The area of the entire figure shown below is $\frac{7}{4}$ of the area of one of the circles in the figure. If the radius of each of the circles is 4 cm, what is the area of overlap? Express your answer as a decimal to the nearest hundredth.



The area of one circle is $4^2(3.14) = 50.24$ square centimeters. Thus, the area of the entire figure must be $(\frac{7}{4})(50.24) = 87.92$ square centimeters. If there was no overlap, the two circles would have a combined area of $2(50.24) = 100.48$ square centimeters. This means that the area of the overlap must be $100.48 - 87.92 = \mathbf{12.56}$ square centimeters, to the nearest hundredth.

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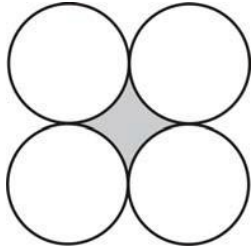
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