# MATHCOUNTS® Problem of the Week Archive

## *The Value of Pi – March 12, 2007*

### *Problems & Solutions*

March 14 is often celebrated as Pi Day because the date can be expressed as 3.14, a numerical approximation for the value of π. March 14, 2007 falls on a Wednesday. At some time in history, humans realized that circles appeared in many sizes and that the greater the distance across a circle the greater the distance around the circle. There is some evidence that about 2000 BC, the Babylonians and the Egyptians knew that the ratio between the distance around a circle (the circumference) and the distance across a circle (the diameter) is a constant. The symbol "π" was first used by William Jones in 1706 to represent the ratio of a circle's circumference to its diameter. The modern representation is $C/d = \pi$.

Around 2000 BC the Babylonians approximated the value of π to be 3 1/8 and the Egyptians approximated the value of π to be $4(8/9)^2$. What is the positive difference between these two approximations for the value of π? Express your answer as a decimal to the nearest thousandth.

*The Babylonians' approximation can be written as 3 1/8 = 3.125, and the Egyptians' approximation can be written as $4(8/9)^2$ = 3.160. So, 3.160 – 3.125 = **0.035**.*

Mary has decided to approximate the value of π by collecting some data. The table below shows her measurements for the length of the circumference and diameter of 5 different circles.

| **Circle** | **A** | **B** | **C** | **D** | **E** |
|---|---|---|---|---|---|
| Circumference (mm) | 89 | 67 | 58 | 35 | 97 |
| Diameter (mm) | 29 | 21 | 18 | 11 | 31 |
| Circumference ÷ Diameter | ? | ? | ? | ? | ? |

What is the mean of the circumference to diameter ratios of these circles? Express your answer as a decimal to the nearest thousandth.

*Circle A: 89 ÷ 29 = 3.069*

*Circle B: 67 ÷ 21 = 3.190*

*Circle C: 58 ÷ 18 = 3.222*

*Circle D: 35 ÷ 11 = 3.182*

*Circle E: 97 ÷ 31 = 3.129*

*Thus, the mean of the circumference to diameter ratios is (3.069 + 3.190 + 3.222 + 3.182 + 3.129) ÷ 5 ≈ **3.158**, to the nearest thousandth.*

The circumference of a circle can be found using the formula $C = \pi d$. The value for π can be approximated by inscribing and circumscribing a polygon with the same number of sides on a circle whose radius ($r$) is 1 unit. The circumference of the circle is greater than the perimeter of the inscribed polygon and less than the perimeter of the circumscribed polygon: Perimeterins poly ≤ $2\pi r$ ≤

Perimetercir poly. Let $2x$ represent the perimeter of a regular hexagon inscribed in a circle with radius 1 and let $2y$ represent the perimeter of a regular hexagon circumscribed about the same circle so that $2x \leq \pi \leq 2y$. What is the mean of $x$ and $y$? Express your answer as a decimal to the nearest hundredth.

*The perimeter of the inscribed hexagon is 6 times the length of one side of an equilateral triangle whose side length is 1:* $6 \times 1 = 6 = 2x$*, so* $x = 3$*. The perimeter of the circumscribed hexagon is 6 times the side length of an equilateral triangle whose height is 1. An equilateral triangle whose height is 1 is part of a 30-60-90 right triangle whose sides are in the ratio of* $a : 2a : a\sqrt{3}$*. The height of the equilateral triangle is* $a\sqrt{3} = 1$*, so* $a = \sqrt{(3)}/3$*. The perimeter of the circumscribed hexagon is* $6 \times (2\sqrt{(3)}/3) \approx 6.928 = 2y$*, so* $y \approx 3.464$*. The mean of x and y is* $(3 + 3.464)/2 =$ ***3.23****, rounded to the nearest hundredth.*

By what percent does the hexagon method overestimate the accepted approximation of 3.14 for the value of $\pi$? Express your answer to the nearest tenth.

*The hexagon method overestimates the approximation of 3.14 for* $\pi$ *by* $(3.23 - 3.14)/3.14 \approx .0287 \approx$ ***2.9%****.*

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