Systems of Equations Stretch

For problems 1-3, solve each of the following systems of equations. Describe the method you used. What other methods could you have used to solve each system? Express each answer as an ordered pair. Express any non-integer value as a common fraction.

1. $2x + 3y = 11$
   $3x - y = 11$

2. $\frac{2}{x} + \frac{3}{y} = 11$
   $\frac{3}{x} - \frac{1}{y} = 11$

3. $2x^2 + 3y^2 = 11$
   $3x^2 - y^2 = 11$

In problems 4 and 5, for what value of k does the linear system have no solution? Express any non-integer value as a common fraction.

4. $kx + 3y = 11$
   $3x - y = 11$

5. $2x + ky = 11$
   $3x - y = 11$

6. Given that $2a + b = 19$, $2c + d = 37$ and $b + d = 24$, what is the value of $a + b + c + d$?

7. Given that $a + b = 29$ and $ab = 204$, what is the value of $a^2 + b^2$?

8. Solve each of the following linear systems. Express your answer as an ordered pair.

a. $5x + 6y = 7$
   $8x + 9y = 10$

b. $x + 2y = 3$
   $4x + 5y = 6$

Many problems can be solved using a linear system of equations. For problems 9 and 10, use linear systems of equations to help you solve.

9a. For each equation below (i, ii, iii and iv), does there exist a solution (x, y) with positive integers x and y? In each case, either find all pairs of integers satisfying the equation or explain why none exist. Hint for the first equation: $x^2 - y^2 = (x + y)(x - y)$, so $x + y = 12$ and $x - y = 4$ is one case to check.

i. $x^2 - y^2 = 48$ ii. $x^2 - y^2 = 23$ iii. $x^2 - y^2 = 45$ iv. $x^2 - y^2 = 90$

b. In general, for what type of integers, n, does $x^2 - y^2 = n$ have at least one solution?

10. A shipping clerk wishes to determine the weights of each of five boxes. Each box weighs a different integer amount less than 100 kg. Unfortunately the only scales available measure weights in excess of 100 kg. The clerk therefore decides to weigh the boxes in pairs so that each box is weighed with every other box. The weights for the 10 pairs of boxes are (in kilograms) 110, 112, 113, 114, 115, 116, 117, 118, 120 and 121. From this information the clerk can determine the weight of each box. What are the weights of each of the five boxes?