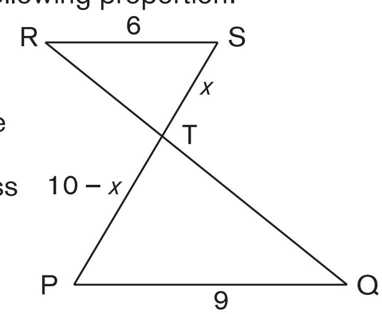
Warm-Up!

1. Triangle ABC is a right triangle with angles of degree measure 45, 45 and 90. Side BC, which is opposite ∠A, must be the same length as side AB, which is oppposite ∠C because we are told that $m\angle A = m\angle C$. So AB = BC = **4**. Using the Pythagorean Theorem, we can determine the length of hypotenuse AC. We have $AC^2 = 4^2 + 4^2 \rightarrow AC^2 = 16 + 16 \rightarrow AC^2 = 32 \rightarrow AC = \sqrt{32} = \mathbf{4\sqrt{2}}$.
NOTE: A property of 45-45-90 right triangles with legs of length *a* is that the hypotenuse has length $a\sqrt{2}$.

2. Since BA:AC = 3:2, it follows that AC = ⅖BC. Therefore, AC = ⅖ × 45 = **18**.

3. Determining triangles RTS and QTP are similar enables us to write the following proportion: $\frac{RT}{QT}=\frac{TS}{TP}=\frac{SR}{PQ}$. Knowing SP = 10, we can let ST = x and then PT = $10 - x$. Going back to our original proportion and filling in some of the known values, we see $\frac{TS}{TP}=\frac{SR}{PQ} \rightarrow \frac{x}{10-x}=\frac{6}{9} \rightarrow \frac{x}{10-x}=\frac{2}{3}$. Using cross products, we have $20 - 2x = 3x \rightarrow 20 = 5x \rightarrow x = 4$. This means ST = **4** units and PT = **6** units.



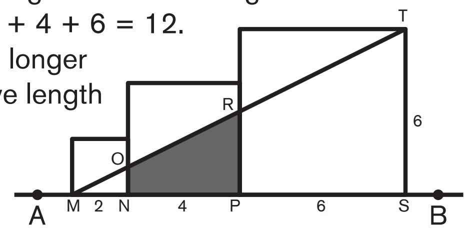
The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

4. Triangle BCD is a 30-60-90 right triangle with a shorter leg of length 6. Based on properties of 30-60-90 right triangles, segment BC, the longer leg, has length $6\sqrt{3}$. Since M is the midpoint of segment AD, $MD = 6\sqrt{3} \div 2 = 3\sqrt{3}$. For right triangle CDM, we know CD = 6 and DM = $3\sqrt{3}$, so we can use the Pythagorean Theorem to determine CM. We have $CM^2 = 6^2 + (3\sqrt{3})^2 \rightarrow CM = \sqrt{(36 + 27)} \rightarrow CM = \sqrt{(63)} \rightarrow CM = 3\sqrt{7}$. If $m\angle DBC = 30°$, then $m\angle BDA = 30°$ because they are alternate interior angles. Also $m\angle CKB = m\angle MKD$ since they are vertical angles. That means △CKB ~ △MKD, and BC/DM = CK/MK. Substituting and simplifying BC/DM, we have 2/1 = CK/MK, which means MK = ⅓CM → MK = ⅓ × $3\sqrt{7}$ → MK = $\mathbf{\sqrt{7}}$.

5. We are told that DE = 2EC, which means that DE/EC = 2/1, and DE = ⅔DC. Since AB = DC, it follows that DE = ⅔AB, and DE/AB = 2/3. Because segments AB and DC are each perpendicular to segment BC, it follows that segment AB and segment CD (or segment DE) are parallel. Thus, $m\angle BAF = m\angle DEF$, and $m\angle FDE = m\angle ABF$ because they are pairs of alternate interior angles. By Angle-Angle Similarity, we have ΔABF ~ ΔEDF. Notice that segment BG is an altitude of ΔABF, and segment CG is the corresponding altitude of ΔEDF. Therefore, CG/BG = 2/3 and BG = ⅗BC. Right triangles BGF and BCD are also similar (Angle-Angle Similarity using the right angles and ∠FBG in each triangle), which means that BC/DC = BG/FG. Substituting and cross-multiplying yields BC/20 = (⅗BC)/FG → BC × FG = 20(⅗BC) → FG = **12**.

6. We are asked to determine the area of the shaded quadrilateral, which happens to be a trapezoid. The height of the trapezoid is 4, the side length of the middle square. Notice that triangles MNO, MPR and MST, shown in the figure, are similar right triangles. The base NO of the trapezoid is the shorter leg of $\triangle$MNO and the base PR is the shorter leg of $\triangle$MPR. Triangle MST has a shorter leg of length ST = 6 and a longer leg of length MS = 2 + 4 + 6 = 12. The ratio of the lengths of the shorter leg to the longer leg is 1:2. The longer leg of $\triangle$MPR has length MP = 2 + 4 = 6, so its shorter leg must have length PR = 1/2 × 6 = 3. The longer leg of $\triangle$MNO has length MN = 2, so its shorter leg must have length NO = 1/2 × 2 = 1. Therefore, the trapezoid has area 1/2 × (1 + 3) × 4 = **8** units^2.



7. If we draw a segment parallel to the x-axis from the upper-right corner of the square to the apex of the triangle, we create a triangle, as shown, with base length 10 + 5 = 15 that is similar to the shaded triangle. The ratio of the sides of the triangles is 10:15 = 2:3. We also know that the combined heights of the two triangles is 10. But these heights also are in the ratio 2:3. Therefore, if we let h represent the height of the shaded triangle, we can write the proportion: $h/(10 - h) = 2/3$. Cross-multiplying, we get $3h = 2(10 - h)$. Solving for h gives us $3h = 20 - 2h \rightarrow 5h = 20 \rightarrow h = 4$. So the height of the shaded triangle is 4 and its base is 10. Therefore, its area is 1/2 × 10 × 4 = **20** units^2.