MATHCOUNTS® Minis September 2018 Activity Solutions

Warm-Up!

1. When we expand the given product, we get $(x + 1)(y + 1) = \mathbf{xy + x + y + 1}$.

2. We are told that $x = y + 3$ and $y = z - 5$, which can be rewritten as $y + 5 = z$. We are asked to determine the value of $z - x$. Substituting we get $(y + 5) - (y + 3) = y + 5 - y - 3 = 5 - 3 = \mathbf{2}$.

3. The problem text can be translated into the equation $((3n + 5) \times 2) - 4 = 36 \rightarrow 6n + 6 = 36$. Solving, we see that $6n = 30$, so $n = \mathbf{5}$.

4. Initially, the bill was split equally between the five friends, meaning each person would pay one-fifth of the total bill. If we will let a represent one-fifth of the total bill, then the total bill equals $5a$. Since the four friends who paid the bill each paid an amount equal to $a + 4$, then the amount of the total bill also equals $4(a + 4) = 4a + 16$. Setting these two expressions equal to each other and solving for a, we get $5a = 4a + 16$, so $a = 16$. Therefore, the total bill was $5 \times 16 = \mathbf{80}$ dollars.

The Problems are solved in the MATHCOUNTS® Minis video.

Follow-up Problems

5. A total of $40 \times 2.15 = \$86$ would have been paid for the forty bowls of chocolate ice cream. The remaining $158.20 - 86 = \$72.20$ would have been paid for bowls of vanilla ice cream. At $\$1.90$ per bowl, that would mean $72.20 \div 1.90 = 38$ bowls of vanilla ice cream were sold. Thus, a total of $40 + 38 = \mathbf{78}$ bowls of ice cream were sold.

6. Let x represent the number of widgets that Marcus has. Then Cindi has $2x$ widgets. Since Kevin has 219 widgets, it follows that Cindi and Marcus have a combined total of $1020 - 219 = 801$ widgets. That means $x + 2x = 801 \rightarrow 3x = 801 \rightarrow x = 267$. So, Marcus has 267 widgets, and Cindi has twice that number of widgets, which is $2 \times 267 = \mathbf{534}$ widgets.

7. Since we don't know the dimensions of the rectangle let's call them L and W. We are told that the rectangle has an area of 108 in^2 which means that $LW = 108$. We are looking for the area if the length and width are each increased by 1. In other words, area $= (L + 1)(W + 1)$. If we expand this expression we get $LW + L + W + 1$. Well we know that $LW = 108$. We are told that the perimeter of the rectangle is 42 which means that $2(L + W) = 42 \rightarrow L + W = 21$. Substituting, we now have $LW + (L + W) + 1 = 108 + 21 + 1 = \mathbf{130}$ in^2.

8. This problem can be solved several ways. First let's solve it algebraically. We are told that Douglas's favorite number is a positive two-digit integer; let's call it AB where A is the tens digit and B is the units digit. That means that the value of his favorite number is 10A + B. Then a new number is created, AB7, where A now is the hundreds digit, B now is the tens digit and 7 is the units digit. The value of the new number is 100A + 10B + 7. Finally, we are told that the new number is 385 more than Douglas's favorite number. So we have 100A + 10B + 7 = 10A + B +385. Subtracting 10A, B and 7 from both sides yields 90A + 9B = 378. Dividing both sides by 9 gives us 10A + B = **42**. This is Douglas's favorite number.