Warm-Up!

1. We are told that \( x = y + 3 \) and \( y = z - 5 \), which can be rewritten as \( y + 5 = z \). We are asked to determine the value of \( z - x \). Substituting we get \((y + 5) - (y + 3) = y + 5 - y - 3 = 5 - 3 = 2\).

2. If we subtract from the total the $30 charged to hook the car to the tow truck, we see that \( 59.75 - 30 = 29.75 \) was the charge for the mileage. So from the school, Mr. Alman's car was towed \( 29.75 \div 1.75 = 17 \) miles to his house.

3. From the information given, we can write the following two equations, where \( x \) represents the weight of Tweedledee and \( y \) is the weight of Tweedledum: \( x + 2y = 361 \) and \( 2x + y = 362 \). Adding the two equations, we get \( 3x + 3y = 723 \). Dividing each side by 3, we see that the sum of their weights is \( x + y = 241 \) pounds.

4. Since we don’t know the dimensions of the rectangle let's call them \( L \) and \( W \). We are told that the rectangle has an area of 108 in\(^2\), which means that \( LW = 108 \). We are looking for the new area after the length and width are each increased by 1. In other words, \((L + 1)(W + 1)\). If we expand this expression we get \( LW + L + W + 1 \). Well, we know that \( LW = 108 \). We are told that the perimeter of the rectangle is 42, which means that \( 2(L + W) = 42 \rightarrow L + W = 21 \). Substituting, we now have \( LW + (L + W) + 1 = 108 + 21 + 1 = 130 \) in\(^2\).

The Problem is solved in the MATHCOUNTS®Mini video.

Follow-up Problems

5. A total of \( 40 \times 2.15 = 86 \) would have been paid for the forty bowls of chocolate ice cream. The remaining \( 158.20 - 86 = 72.20 \) would have been paid for bowls of vanilla ice cream. At $1.90 per bowl, that would mean \( 72.20 \div 1.90 = 38 \) bowls of vanilla ice cream were sold. Thus, a total of \( 40 + 38 = 78 \) bowls of ice cream were sold.

6a. We are told that the perimeter of the painting is 48 inches. Since adding a frame that results in a one-inch margin around the painting essentially adds an additional 2 inches at each corner of the painting, the outer perimeter of the frame is \( 48 + 8 = 56 \) in.

6b. We are told that the perimeter of the painting is 48 inches. That means \( 2L + 2W = 48 \). As the figure shows, the area of the frame is the sum of the areas of the \( 1 \times L \) regions at the top and bottom of the painting, the \( 1 \times W \) regions on either side and the \( 1 \times 1 \) regions at each of the four corners. Thus, the area is of the frame is \( 2L + 2W + 4 = 48 + 4 = 52 \) in\(^2\).

7. Let \( p \) represent the number of pit bulls, \( c \) is the number of chihuahuas and \( m \) is the number of mutts. The second sentence of the problem yields the following equations, where \( A \) is the total number of dogs: \( p = A - 23 \), \( c = A - 17 \), \( m = A - 28 \) and \( A = p + c + m \). If we add the first three equations we get \( p + c + m = 3A - 68 \). Substituting, we get \( A = 3A - 68 \). We now solve to determine that the total number of dogs at the pound is \( 2A = 68 \rightarrow A = 34 \) dogs.
8. This problem can be solved several ways. First let's solve it algebraically. We are told that Douglas' favorite number is a positive two-digit integer; let's call it AB where A is the tens digit and B is the units digit. That means that the value of his favorite number is 10A + B. Then a new number is created, AB7, where A now is the hundreds digit, B now is the tens digit and 7 is the units digit. The value of the new number is 100A + 10B + 7. Finally, we are told that the new number is 385 more than Douglas' favorite number. So we have 100A + 10B + 7 = 10A + B + 385. Subtracting 10A, B and 7 from both sides yields 90A + 9B = 378. Dividing both sides by 9 gives us 10A + B = 42. This is Doug's favorite number.

We could also have solved the problem logically by setting up the vertical addition problem:

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  3 8 5
+ A B
  A B 7
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Notice that 5 + B = 7, so B must equal 2. We can then substitute 2 for B in the problem to get:

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  3 8 5
+ A 2
  A 2 7
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The only integer from 1 to 9 that yields a units digit of 2 when added to 8 is 4. It follows that:

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  3 8 5
+ 4 2
  4 2 7
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Thus, Douglas’ favorite number is 42.