Warm-Up!

1. If we subtract the second equation from the first equation we get

$$u + v + w + x + y + z = 45$$
$$- (v + w + x + y + z = 21)$$
$$u = \mathbf{24}$$

2. From the information given, we can write the following two equations, where x represents the weight of Tweedledee and y is the weight of Tweedledum: $x + 2y = 361$ and $2x + y = 362$. Adding the two equations, we get $3x + 3y = 723$. Dividing each side by 3, we see that the sum of their weights is $x + y = \mathbf{241}$ pounds.

3. Let x represent the first number and y represent the second number. We are told that $x + y = 6$ and $xy = 7$. We are asked to find the sum of the reciprocals of the two numbers, $1/x + 1/y$, which can be rewritten as $(y + x)/xy$. Substituting, we have $(y + x)/xy = \mathbf{6/7}$.

4. The combined length of segments AB and BC is the length of AC, so we have $(2x + 5) + (6x − 1)$ $= 36$. Combining like terms, we get $8x + 4 = 36$. Dividing both sides by 4, we get $2x + 1 = 9$. Adding 4 to each side, we get $2x + 5 = 13$. Thus, AB = **13** cm.

The Problems are solved in the **MATHCOUNTS**®*Mini* video.

Follow-up Problems

5. Let p represent the number of pit bulls, c is the number of chihuahuas and m is the number of mutts. The second sentence of the problem yields the following equations, where A is the total number of dogs: $p = A − 23$, $c = A − 17$, $m = A − 28$ and $A = p + c + m$. If we add the first three equations we get $p + c + m = 3A − 68$. Substituting, we get $A = 3A − 68$. We now solve to determine that the total number of dogs at the pound is $2A = 68 \rightarrow A = \mathbf{34}$ dogs.

6. Each of the five sums of the ages of each group of four is the sum of all five ages minus one friend. If the friends' ages are represented by variables a, b, c, d and e. Then we have $a + b + c + d = 58$, $a + b + c + e = 59$, $a + b + d + e = 61$, $a + c + d + e = 62$ and $b + c + d + e = 64$. Adding these five equations together, we get $4(a + b + c + d + e) = 304$. The sum of all five ages is, therefore, $304/4 = 76$. We are looking for the age of the oldest friend. The smallest of the four sums will be the sum in which the oldest friend was not counted. Subtracting this sum from the sum of all five, we get $76 − 58 = \mathbf{18}$.

7. We are told that $xyz = 45$ and $1/x + 1/y + 1/z = 1/5$. We can rewrite the left side of the second equation using a common denominator to get $(yz/xyz) + (xz/xyz) + (xy/xyz) = 1/5 \rightarrow$ $(xy + xz + yz)/xyz = 1/5$. But we know that $xyz = 45$, so we have $(xy + xz + yz)/45 = 1/5 \rightarrow$ $xy + xz + yz = 9$. If the sum of the three products xy, xz and yz is 9, then their mean is $9/3 = \mathbf{3}$.

8. If we use the variables l and w to represent length and width of the rectangle, respectively, we can write the equations $lw = 168$ and $2l + 2w = 62$. We are looking for the product of the length of the diagonals, which is $l^2 + w^2$. Taking our perimeter equation and dividing by 2, we get $l + w = 31$. Squaring both side, yields $l^2 + 2lw + w^2 = 961$. Substituting 168 in for lw, we get $l^2 + 2(168) + w^2$ $= 961$. So, $l^2 + w^2 = 961 − 2(168) = \mathbf{625}$.