# MATHCOUNTS® Minis October 2015 Activity Solutions

## Warm-Up!

1. Because these are each arithmetic sequences, we know the difference between consecutive terms for each sequence remains constant. (In other words, the same amount is added to each term to get the next term.)

   (a) The common difference is 11 – 5 = 6. The terms of the sequence are 5, 11, *11 + 6 =* **17**, *17 + 6 =* **23**, *23 + 6 =* **29**.

   (b) Again, the common difference is 6, but we must work backwards. __, __, *5 – 6 =* **–1**, 5, 11. Then __, *–1 – 6 =* **–7**, **–1**, 5, 11. And finally *–7 – 6 =* **–13**, **–7**, **–1**, 5, 11.

   (c) Going from 5 to 11, we must add the common difference to 5 a total of four times. The total difference is 6. Dividing this into four equal parts, we see the common difference of is 6/4 = 3/2 = 1.5. Therefore, the sequence is 5, **6.5**, **8**, **9.5**, 11.

2. The common difference of this arithmetic sequence is 33 – 31 = 2, since 31 and 33 are consecutive terms of the sequence. Because this sequence has an even number of terms, 14, there will be the first 14 ÷ 2 = 7 terms and the second 7 terms. Therefore, 31 is the 7th term and 33 is the 8th term. To get from the 7th term to the 14th term, we need to add the common difference 7 times: 31 + 7(2) = **45**.

3. To get from the 1st term of this sequence to the 5th term, we must add the common difference to the 1st term 4 times. Thus, the 5th term of the sequence is 8 + 3(4) = **20**. To get from the 1st term to the 35th term, we must add the common difference to the 1st term 34 times, 8 + 3(34) = **110**.

4. The $n$th term of an arithmetic sequence where $a$ is the 1st term of the sequence and $d$ is the common difference can be represented as $\boldsymbol{a + d(n - 1)}$.

To test our expression we can plug in the values from the previous problem and see if we get the same answers. Our common difference, $d$, is 3 and our first term, $a$, is 8. If we are looking for the 5th term, we will plug 5 into the equation for $n$:
$8 + 3 \times (5 - 1) = 8 + 3 \times 4 = 8 + 12 = 20$.
If we are looking for the 35th term we plug 35 into the equation for $n$:
$8 + 3 \times (35 - 1) = 8 + 3 \times 34 = 8 + 102 = 110$.
Our answers match what we found in the previous problem.

**The Problems** are solved in the MATHCOUNTS® Mini video.

## Follow-up Problems

5. We can ignore the unknown first term and consider the sequence beginning with –7 and with 106 as the fourth term. We know that to go from –7 to 106 the common difference, $d$, is added three times. So we have the equation $-7 + 3d = 106$. Solving this equation we see that $3d = 113$, and $d = 113/3$. The fourteenth term of the original sequence is now the thirteenth term when we disregard the first term and consider the sequence beginning with the second term, –7. So the thirteenth term of the sequence, begining with –7 is:
$-7 + 12d = -7 + 12 \times (113/3) = -7 + 4 \times 113 = -7 + 452 =$ **445**.

6. We are told that $y/w = 3$, so $y = 3w$. We have the sequence $v$, $w$, $x$, $3w$, $z$. Since this is an arithmetic sequence, it follows that $x$ must be $2w$ (since it's the average of its two neighboring terms), the common difference is $w$, and $z$, then, is $4w$. That means that $z/x = (4w)/(2w) =$ **2**.

7. If the sum of the first five terms of an arithmetic sequence is 75, then the middle term, or 3rd term, is $75 \div 5 = 15$. The extended sequence with 11 terms must have a middle term, or 6th term, of $363 \div 11 = 33$. The difference between the 6th term and 3rd term is $33 - 15 = 18$, which is the common difference added 3 times. So the common difference is $18 \div 3 = 6$. Going from the 3rd term to the 1st term would mean subtracting the common difference twice: $15 - 2(6) =$ **3**.

8. The sum of the interior angles of an octagon is 1080 degrees. Being that there are 8 angles, the average/middle value must be $1080 \div 8 = 135$. Given that the problem says this is an increasing sequence, we know we can't have an equiangular octagon with 8 angles each measuring 135 degrees. If our 4th term is smaller than the average by 1, the common difference is 2. We need to ensure the 4th term minus three of these differences is positive. Additionally, we need to ensure the 5th term plus three of these differences is less than 180 (to keep the figure convex). This upper limit is what we need to worry about (since 135 is closer to 180 than 0). We see that having 4th and 5th terms of 134 and 136; 133 and 137; and so on will work up to a certain point. Where is that point? If we add $x$ to 135 to get the 5th angle, then the common difference of the sequence is $2x$. Then the 6th angle is $135 + 3x$, the 7th angle is $135 + 5x$ and the 8th angle is $135 + 7x$ (which we see is also the 5th angle plus 3 of these differences of $2x$). We need $135 + 7x < 180$. Subtracting 135 from both sides and then dividing by 7 yields $x < 6.43..$, so $x = 6$ is the largest value that works. Remember, this was the amount to add to 135 to get the 5th angle. Therefore, we could add 1, 2, 3, 4, 5 or 6, and there must be **6** possible sequences.