

Warm-Up!

1. The area of the shaded region is the difference of the areas of the square and the circle. Since the square has sides of length 10 units, it follows that its area is $s^2 = 10^2 = 100$ units². The diameter of the circle is also 10 units, which means the radius is 5 units. The area of the circle then is $\pi r^2 = \pi(5^2) = 25\pi$ units². Therefore, the area of the shaded region is $100 - 25\pi$ unit² \approx **21.46** units².

2. The area of the shaded region is the difference of the areas of the circle and the square. Since the square has sides of length 10 units, it follows that its area is $s^2 = 10^2 = 100$ units². The diagonal of the square is a diameter of the circle. The diagonal of the square is also the hypotenuse of a 45-45-90 right triangle. Based on the properties of 45-45-90 right triangles, we know the length of the diagonal, and therefore the circle's diameter is $10\sqrt{2}$ units. This means the radius of the circle is $5\sqrt{2}$ units, and its area is $\pi r^2 = \pi(5\sqrt{2})^2 = 50\pi$ units². We see now that the area of the shaded region is $50\pi - 100$ units² \approx **57.08** units².

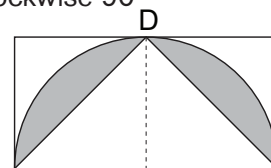
3. The area of rectangle ABCD is $27(11) = 297$ units². If we subtract the area of the triangular region that is removed from the area of rectangle ABCD, the result is the area of pentagon ABEFD. Now $CF = CD - FD = 27 - 15 = 12$ units, and $EC = BC - BE = 11 - 6 = 5$ units. Thus the area of $\triangle CEF$ is $\frac{1}{2}(12)(5) = 30$ units². That means the area of pentagon ABEFD is $297 - 30 =$ **267** units².

4. We are told that segment XY intersects the center of the square, which means $CY = AX = 2$ units. We can now determine that $DY = DC - YC = 10 - 2 = 8$ units. Now quadrilateral AXYD is a trapezoid with bases DY and AX. The area of a trapezoid is just the average of the lengths of its bases times its height. Thus, the area of trapezoid AXYD is $\frac{1}{2}(2 + 8)(10) = \frac{1}{2}(10)(10) = 5(10) =$ **50** units².

The Problem is solved in the **MATHCOUNTS**® *Mini*® video.

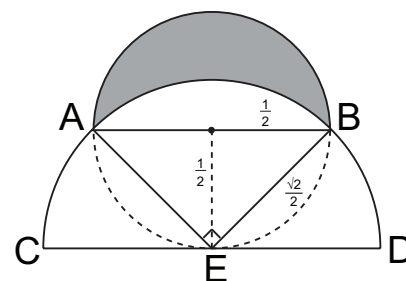
Follow-up Problems

5. If we slice the square along diagonal BD and rotate the half containing $\triangle ABD$ clockwise 90° about the original point D, the result is the figure shown here. We can see now that the area of the shaded region is just the area of the semicircle with radius 8 units less the area of the triangle with base and height measuring 16 units and 8 units, respectively. The area of the semicircle is $\frac{1}{2} \times \pi \times r^2 = \frac{1}{2} \times \pi \times 8^2 = 32\pi$ units². The area of the triangle is $\frac{1}{2} \times b \times h = \frac{1}{2} \times 16 \times 8 = 64$ units². That means the area of the shaded region is $32\pi - 64$ units² \approx **36.53** units².



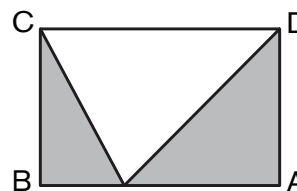
6. The area of the shaded region is the area of the equilateral triangle less the area of the sector of the circle with central angle measuring 60° . Recall that using the properties of 30-60-90 right triangles we can conclude that the height of the equilateral triangle is $3\sqrt{3}$ units. Therefore, the area of the triangle is $\frac{1}{2} \times b \times h = \frac{1}{2} \times 6 \times 3\sqrt{3} = 9\sqrt{3}$ units². The area of the sector is $60/360 = 1/6$ the area of the circle with a radius of $3\sqrt{3}$ units. The area of the sector then is $1/6 \times \pi \times r^2 = 1/6 \times \pi \times (3\sqrt{3})^2 = (27/6)\pi = (9/2)\pi$ units². Therefore, the area of the shaded region is $9\sqrt{3} - (9/2)\pi$ units² \approx **1.45** units².

7. The area of the lune is the area of the smaller semicircle less the area of the segment of the larger semicircle intercepting arc AB and bounded by chord AB. Since the $AB = 1$, the area of the smaller semicircle is $\frac{1}{2} \times \pi \times r^2 = \frac{1}{2} \times \pi \times (\frac{1}{2})^2 = \frac{\pi}{8}$ units². The area of the segment of the larger semicircle bounded by chord AB is the area of sector of the larger semicircle intercepting arc AB less the area of $\triangle ABE$, as shown, where E is the midpoint of segment CD. Notice that $BE = \frac{\sqrt{2}}{2}$ because it is a radius of the larger semicircle whose diameter we are told is $\sqrt{2}$ units.

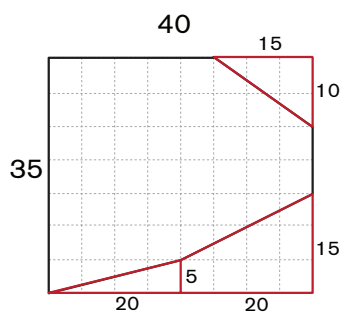


The segment drawn from point E perpendicular to segment AB intersects the center of the smaller semicircle. Using the Pythagorean Theorem we see that the length of this segment is $\frac{1}{2}$ units, which means point E also lies on the smaller circle. Since $\angle AEB$ intercepts the diameter of the smaller semicircle, $m\angle AEB = 90^\circ$. That means the area of the sector intercepting arc AB is $\frac{90}{180} = \frac{1}{2}$ the area of the semicircle. The area of the sector is $\frac{1}{2} \times (\frac{1}{2} \times \pi \times r^2) = \frac{1}{2} \times (\frac{1}{2} \times \pi \times (\frac{\sqrt{2}}{2})^2) = \frac{1}{2} \times (\frac{1}{4}\pi) = \frac{\pi}{8}$. The area of $\triangle AEB$ is $\frac{1}{2} \times b \times h = \frac{1}{2} \times 1 \times \frac{1}{2} = \frac{1}{4}$ units². It follows that the area of the segment is $\frac{\pi}{8} - \frac{1}{4}$. Therefore, the area of the lune is $\frac{\pi}{8} - (\frac{\pi}{8} - \frac{1}{4}) = \frac{\pi}{8} - \frac{\pi}{8} + \frac{1}{4} = \frac{1}{4}$ units².

8. The figure appears to be a rectangle from which side CD has been removed and two of the three interior triangles have been shaded, as shown. If we determine the area of rectangle ABCD and subtract from it the area of the unshaded triangle, the result will be the area of the shaded region. The area of the rectangle is $(8)(12) = 96$ cm². The area of the unshaded triangle is $\frac{1}{2}(12)(8) = (6)(8) = 48$ cm². That means the area of the shaded region is $96 - 48 = 48$ cm².



You may also have recognized that if we rotate the two shaded triangles 180° and translate them so that segment BC and segment AD perfectly overlap, the resulting figure fits the unshaded region exactly. The combined area of the two shaded triangles is the same as the area of the unshaded triangle with base = 12 cm and height = 8 cm.



7. To find the area of this irregular-shaped region, let's first determine the area of the large rectangular region. The area is $40(35) = 1400$ ft². Next, we can break up the area outside the irregular shape, but inside the rectangular region, into several shapes with easily determined areas, as shown. The triangle in the upper right corner of the rectangular region has an area of $\frac{1}{2}(10)(15) = 75$ ft². The triangle in the lower left corner has an area of $\frac{1}{2}(20)(5) = 50$ ft². The area of the trapezoid in the lower right corner is $\frac{1}{2}(5 + 15)(20) = \frac{1}{2}(20)(20) = 200$ ft². That means the area of the irregular-shaped region is $1400 - (75 + 50 + 200) = 1400 - 325 = 1075$ ft².