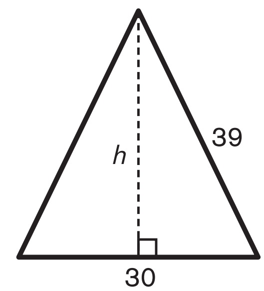
MATHCOUNTS® Minis November 2016 Activity Solutions

Warm-Up!

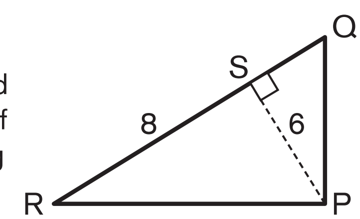


1. We are told the triangle has base length 30, so we just need the height of the triangle to determine its area. The altitude of this triangle drawn from the vertex to the base creates two congruent right triangles, as shown. We know that the length of the hypotenuse of each of these triangles is 39, and the length of the shorter leg is 1/2 × 30 = 15. We can now use the Pythagorean Theorem to find the height, h, of the isosceles triangle. We have $15^2 + h^2 = 39^2 \rightarrow 225 + h^2 = 1521 \rightarrow h^2 = 1296 \rightarrow h = 36$. Therefore, the area of the isosceles triangle, in square units, is 1/2 × 30 × 36 = **540**.

2. Since segments MN and OP are parallel, we can conclude that ΔMNQ ~ ΔPOQ (Angle-Angle). Therefore, the ratios of corresponding sides of the triangles are congruent. Since ON = 24 units, it follows that OQ = 24 – QN. We can set up the following proportion: QN/(24 – QN) = 12/20. Cross-multiplying and solving for QN, we get 20(QN) = 12(24 – QN) → 20(QN) = 288 – 12(QN) → 32(QN) = 288 → QN = **9** units.

3. Since ΔSPQ ~ ΔSTU (Angle-Angle), the ratios of corresponding sides of the triangles are congruent. We are told that SP = 2PT → ½(SP) = PT. Since ST = SP + PT, we can write ST = SP + ½(SP) → ST = 3/2(SP) → SP/ST = 3/2. Since the ratio of corresponding sides of the triangles is 2/3, the ratio of the area of ΔSPQ to the area of ΔSTU is $2^2/3^2$ = 4/9. We also are told that the area of ΔSTU is 45 cm^2. So, it follows that the area of ΔSPQ = 4/9(45) = **20** cm^2.

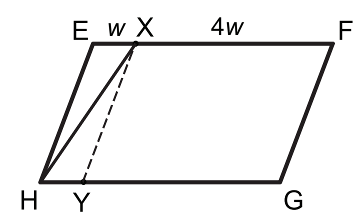
4. Segment PS is an altitude of ΔPQR drawn perpendicular to the hypotenuse, as shown. When an altitude is drawn to the hypotenuse of a right triangle, the two triangles formed are similar to each other and to the original right triangle. Therefore, ΔPQR ~ ΔSQP ~ ΔSPR. Since these triangles are similar, the ratios of the lengths of corresponding sides are equal. So we can write the proportion PS/SR = QP/PR. We are told that PS = 6 and SR = 8, which means PR = 10 (side lengths are a multiple of the Pythagorean Triple 3-4-5). Substituting these values and cross-multiplying yields 6/8 = PQ/10 → 8(PQ) = 6 × 10 → PQ = 60/8 = **15/2**.



The Problems are solved in the MATHCOUNTS® Mini video.

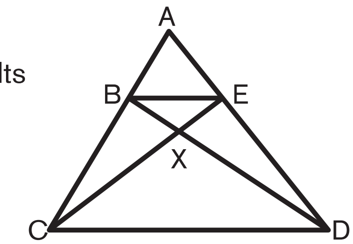
Follow-up Problems

5. We are asked to determine the ratio of the area of ΔEXH to the area of parallelogram EFGH. If we draw a point Y on side GH such that GH = 5YH, parallelogram EXYH is created. The area of ΔEXH is 1/2 the area of parallelogram EXYH. The area of parallelogram EXYH is 1/5 times the area of parallelogram EFGH. Therefore, $\frac{[\Delta EXH]}{[\square EFGH]} = \frac{1}{2} \times \frac{1}{5} = \mathbf{\frac{1}{10}}$.



6. Since QR = QU + UR and we are told that QR = 4, we have 4 = QU + UR → UR = 4 – QU. For similar triangles PQR and TUR, we can write the following proportion: 4/(4 – QU) = 3/UT. Because QSTU is a square, it follows that QS = QU = UT = ST. Substituting, we get 4/(4 – QU) = 3/QU. Cross-multiplying and solving, we see that 4(QU) = 3(4 – QU) → 4(QU) = 12 – 3(QU) → 7(QU) = 12 → QU = ST =**12/7** units.

7. From the figure, we can see that the area of ΔACD is the sum of the areas of ΔABE and trapezoid BCDE. Also, we are told that the area of trapezoid BCDE is 8 times the area of ΔABE. It follows that the area of ΔACD is 9 times the area of ΔABE. That means the ratio of sides BE and CD is $\sqrt{1}/\sqrt{9} = 1/3$. Since segments BE and CD are also sides of triangles EBX and CDX, respectively, it follows that the ratio of the areas of ΔEBX and ΔCDX is $1^2/3^2 = 1/9$. The problem states that the area of ΔCDX is 27 units^2, so the area of ΔEBX is $(1/9) \times 27 = 3$ units^2. We can determine the areas of ΔBCX and ΔDEX by multiplying $\sqrt{3} \times \sqrt{27} = \sqrt{81} = 9$. Therefore, ΔBCX and ΔDEX each have an area of 9 units^2. We now can calculate the area of trapezoid BCDE to be $3 + 27 + 9 + 9 = 48$ units^2. Using Harvey's trick results in the same answer since $(\sqrt{3} + \sqrt{27})^2 = (\sqrt{3} + 3\sqrt{3})^2 = (4\sqrt{3})^2 = 48$ units^2. So the area of ΔABE is $(1/8) \times 48 = 6$ units^2. Thus, the area of ΔACD is $48 + 6 =$ **54** units^2. This also confirms our assertion that the area of ΔACD is 9 times the area of ΔABE since $9 \times 6 =$ **54** units^2.



8. We are told that DE = 2EC, which means that DE/EC = 2/1, and DE = (2/3)DC. Since AB = DC, it follows that DE = (2/3)AB, and DE/AB = 2/3. Because segments AB and DC are each perpendicular to segment BC, it follows that segment AB and segment CD (or segment DE) are parallel. Thus, $m\angle BAF = m\angle DEF$, and $m\angle FDE = m\angle ABF$ because they are pairs of alternate interior angles. By Angle-Angle Similarity, we have $\Delta ABF \sim \Delta EDF$. Notice that segment BG is an altitude of ΔABF, and segment CG is the corresponding altitude of ΔEDF. Therefore, CG/BG = 2/3 and BG = (3/5)BC. Right triangles BGF and BCD are also similar (Angle-Angle Similarity using the right angles and $\angle FBG$ in each triangle), which means that BC/DC = BG/FG. Substituting and cross-multiplying yields $BC/20 = ((3/5)BC)/FG \rightarrow BC \times FG = 20((3/5)BC) \rightarrow FG =$ **12**.