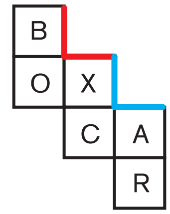
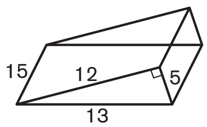
MATHCOUNTS® Minis — May 2017 Activity Solutions

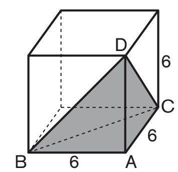
Warm-Up!



1. We see that O and C share a side with face X. When folded, B and A will also share a side with face X. This means that O, C, B and A are not opposite face X. The opposite face to X must be face **R**.



2. When we fold along the dashed lines, we obtain the triangular prism shown here. The volume is the area of the triangle multiplied by the depth of the prism or 1/2 × 5 × 12 × 15 = **450** in^3.



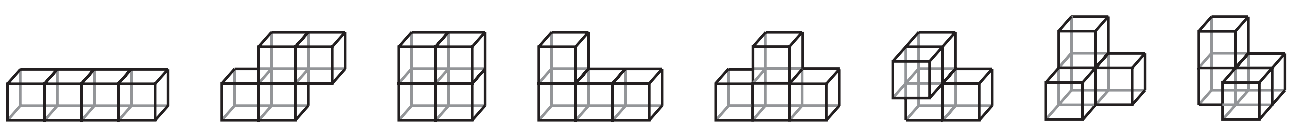
3. Tetrahedron ABCD is shown here. The volume of the tetrahedron (which is a pyramid with a triangular base) is 1/3 × *B* × *h*, where *B* is the area of the base, and *h* is the height of the tetrahedron. The area of the base (ΔABC) is 1/2 × 6 × 6 = 18 cm^2. Since the height is 6 cm, the volume of the tetrahedron is 1/3 × 18 × 6 = **36** cm^3.

4. The volume of a cylinder is the area of the circular base times the height. If we want the two volumes to be equal, we can set up the following equation $\pi \times (3/2)^2 \times 6 = \pi \times (4/2)^2 \times h$. Solving for *h*, we get $h = 3^2 \times 6 \div 4^2 = 54/16 =$ **3 3/8** inches.

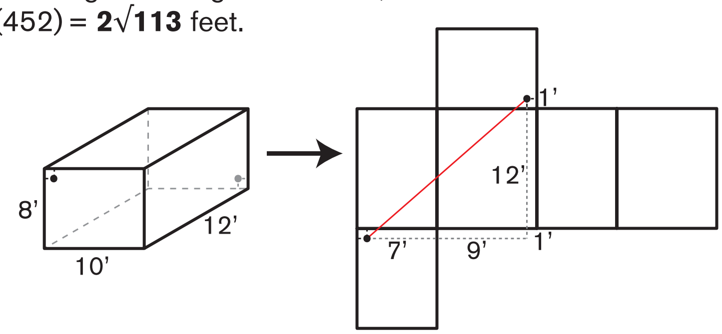
The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

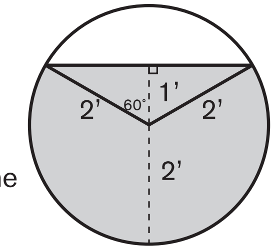
5. As shown, **8** geometric solids are possible.



6. If we open up the box, we can measure the shortest distance from the gecko to the fly as the hypotenuse of a right triangle with legs 16 and 14, as shown. So the distance is $\sqrt{(16^2 + 14^2)} = \sqrt{(256 + 196)} = \sqrt{(452)} =$ **$2\sqrt{113}$** feet.



7. The figure shows the area of the circular base, when the cylinder is on its side, that is covered by the oil. The volume of the oil, will be this area multipled by the height of the tank when upright. In order to find this area, we need to find the area of the segment of the circle not covered by oil and subtract this from the area of the entire circle. Drawing in our known distances and a few radii, we can see find the angle of the arc of the segment. The perpendicular distance from the center of the circle to the top of the oil is 1 foot, and the radius is 2 feet. We see a right triangle with one side length of 1 and the hypotenuse of length 2. The other leg will be $\sqrt{3}$, these are the side ratios of a 30-60-90 triangle. So the entire angle of the arc is $2 \times 60 = 120$ degrees. The area of the arc is 1/3 of the area of the circle, since 120 degrees is 1/3 of the entire measure of a circle. The area of the segment is the area of the arc minus the area of the triangle or $1/3 \times \pi \times 2^2 - 1/2 \times 2\sqrt{3} \times 1$. This we subtract from the entire area of the circle to get $\pi \times 2^2 - (1/3 \times \pi \times 2^2 - 1/2 \times 2\sqrt{3} \times 1) = 2/3 \times \pi \times 2^2 + 1/2 \times 2\sqrt{3} \times 1 \approx 10.1$ square feet. The volume is $15 \times 10.1 = 151.5$ cubic feet. We are looking for the height of the oil when the cylinder is upright. We will set up the following equation: $151.5 = \pi \times 2^2 h$. Solving for h, we get $h \approx$ **12.1** feet.



8. The polyhedron is a cube from which two corners have been removed. The length, width and height of the prism are each 4 units. We can determine the volume of the polyhedron using the method demonstrated in the video by finding the difference of the volume of the cube and the combined volume of the two corners that have been removed. The volume of the cube is $l \times w \times h = 4 \times 4 \times 4 = 64$ units^3. The volume of each tetrahedral corner that has been removed is $1/3 \times B \times h = 1/3 \times (½ \times 4 \times 4) \times 4 = 1/3 \times 8 \times 4 = 32/3$ units^3. Therefore, the area of the polyhedron is $64 - (2 \times 32/3) = 64 - (64/3) =$ **128/3** units^3 $\approx$ **42.67** units^3.