MATHCOUNTS® Minis

March 2017 Activity Solutions

Warm-Up!

1. If $f(n) = n^2 + n + 17$, then to find $f(11)$ we simply need to substitute in 11 for each n that appears in the equation. Doing so, we find that $f(11) = 11^2 + 11 + 17 = 121 + 11 + 17 =$ **149**.

2. If $S(n)$ is a function that returns the sum of the first n positive integers, then $S(20)$ is the sum of the first 20 integers, $1 + 2 + 3 + \ldots + 19 + 20$, and $S(19)$ is the sum of the first 19 integers, $1 + 2 + 3 + \ldots + 19$. The difference is $S(20) - S(19) =$ **20**.

3. Knowing the distance is 64 feet, to find the time it will take to fall, we must substitute 64 into the equation for d and solve. We get $64 = 16t^2 \rightarrow 4 = t^2 \rightarrow t =$ **2** seconds.

4. Similar to the previous problems, we need to substitute r for the variables a and b and 3 for the variable c. Doing so yields $r \times r^3 = 625 \rightarrow r^4 = 625 = 25 \times 25 = 5 \times 5 \times 5 \times 5 \rightarrow r =$ **5**.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. Let's start by substituting the values $x = 0$ and $y = 8$ into the equation to find the constant, c. We get $8 = c \cdot 2^0$ or $c = 8$. Now we can solve for y when $x = 2$. We get $y = 8 \cdot 2^2 = 8 \cdot 4 =$ **32**.

6. First, we should substitute $g(x) = 108$ into the equation to solve for $f(x)$. We get $108 = 2(f(x))$ or $f(x) = 54$. Next, we can solve for the value of x: $54 = x^2 + 5 \rightarrow 49 = x^2 \rightarrow x = \pm 7$. So, the greatest possible value of $f(x + 1) = f(7 + 1) = f(8) = 8^2 + 5 = 64 + 5 =$ **69**.

7. If $f(3m) = 3(f(m))$, then $(3m)^2 + 12 = 3(m^2 + 12) \rightarrow 9m^2 + 12 = 3m^2 + 36 \rightarrow 6m^2 = 24 \rightarrow m^2 = 4 \rightarrow m =$ **2**.

8. Let's count the squares of the stages shown to see what pattern is emerging. Stage 1 starts with 1 square. From stage 1 to stage 2, 4 squares are added. From stage 2 to stage 3, 8 squares are added. From stage 3 to stage 4, 12 squares are added. The number of squares is increasing following the arithmetic sequence $1 + 4 \cdot 1 + 4 \cdot 2 + 4 \cdot 3 + \ldots. + 4 \cdot (n - 1)$, where n is the stage number. At stage 10, there will be $1 + 4 \cdot 1 + 4 \cdot 2 + \ldots + 4 \cdot 9 = 1 + 4(1 + 2 + \ldots + 9) = 1 + 4(45) = 1 + 180 =$ **181**.