MATHCOUNTS® Minis — March 2016 Activity Solutions

Warm-Up!

1. Solve the equation $\sqrt{(x + 4)} = 3$ by squaring both sides. We get $x + 4 = 9$, so $x =$ **5**.

2. When we expand the given product we get $(a + b)(a + b) = a^2 + ab + ab + b^2 =$ $\boldsymbol{a^2 + 2ab + b^2}$.

3. If we subtract the second equation from the first equation we get

$$\begin{array}{rl} u + v + w + x + y + z & = 45 \\ \underline{-\ (v + w + x + y + z} & \underline{= 21\ )} \\ u & = \mathbf{24} \end{array}$$

4. Let x represent the first number and y represent the second number. We are told that $x + y = 6$ and $xy = 7$. We are asked to find the sum of the reciprocals of the two numbers, $1/x + 1/y$, which can be rewritten as $(y + x)/xy$. Substituting, we have $(y + x)/xy =$ **6/7**.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. Since we ultimately want the product of the three integers, let's start by multiplying the products of the pairs we are given. We have $(xy)(yz)(xz) = 4 \times 18 \times 50 \rightarrow x^2y^2z^2 = 3600 \rightarrow (xyz)^2 = 3600$. Now if we take the square root of each side, we get $xyz =$ **60**, since x, y and z are positive numbers.

6. We are told that $xyz = 45$ and $1/x + 1/y + 1/z = 1/5$. We can rewrite the left side of the second equation using a common denominator to get $(yz/xyz) + (xz/xyz) + (xy/xyz) = 1/5 \rightarrow$ $(xy + xz + yz)/xyz = 1/5$. But we know that $xyz = 45$, so we have $(xy + xz + yz)/45 = 1/5 \rightarrow$ $xy + xz + yz = 9$. If the sum of the three products xy, xz and yz is 9, then their mean is $9/3 =$ **3**.

7. Let's start by cubing each side of the given equation to get

$$\left(a + \frac{1}{a}\right)^3 = 3^3$$

$$\left(a + \frac{1}{a}\right)^2\left(a + \frac{1}{a}\right) = 27$$

$$\left(a^2 + 2 + \frac{1}{a^2}\right)\left(a + \frac{1}{a}\right) = 27$$

$$a^3 + a + 2a + \frac{2}{a} + \frac{1}{a} + \frac{1}{a^3} = 27$$

$$a^3 + 3a + \frac{3}{a} + \frac{1}{a^3} = 27.$$

If we factor a 3 out of the two middle terms of the expression on the left-hand side of the equation, we have

$$a^3 + 3\left(a + \frac{1}{a}\right) + \frac{1}{a^3} = 27.$$

Since we know that $a + \dfrac{1}{a} = 3$, we can substitute and simplify to get

$$a^3 + 3(3) + \frac{1}{a^3} = 27$$

$$a^3 + 9 + \frac{1}{a^3} = 27$$

$$a^3 + \frac{1}{a^3} = \mathbf{18}.$$

8. First, let's multiply the two given equations together

$$\left(x + \frac{1}{y}\right)\left(y + \frac{1}{z}\right) = 2\left(\frac{1}{2}\right)$$

$$xy + \frac{x}{z} + 1 + \frac{1}{yz} = 1$$

$$xy + \frac{x}{z} + \frac{1}{yz} = 0.$$

Multiplying through by z, we get the equation

$$xyz + x + \frac{1}{y} = 0.$$

Substituting in and solving for xyz, we find that

$$xyz + 2 = 0$$

$$xyz = \mathbf{-2}.$$