MATHCOUNTS® Minis January 2017 Activity Solutions

Warm-Up!

1. Using the F.O.I.L. method to multiply the binomials, we get $(x + 2)(x - 7) = x^2 - 7x + 2x - 14$. Once we combine like terms, our answer is $\mathbf{x^2 - 5x - 14}$.

2. Since $x = 3$ and $x = -4/3$ are the solutions to $kx^2 - 5x - 12 = 0$, we know $x - 3 = 0$ or $3x = -4 \rightarrow 3x + 4 = 0$. The quadratic, then, can be written as the product of the two binomials $x - 3$ and $3x + 4$ to get $(x - 3)(3x + 4) = 3x^2 + 4x - 9x - 12 = 3x^2 - 5x - 12$. We now can see that $k = \mathbf{3}$.

3. $4^{10} = (2^2)^{10} = 2^{20}$ and $8^{20} = (2^3)^{20} = 2^{60}$, so $4^{10} \times 8^{20} = 2^{20} \times 2^{60} = 2^{20+60} = \mathbf{2^{80}}$.

4. $b(b^4 \times b^3)^2 = b(b^{4+3})^2 = b(b^7)^2 = b(b^{14}) = b^{15} = b^{3(5)}$, so $x = \mathbf{5}$.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. Let's start by squaring each side of the given equation to get

$$\left(a + \frac{1}{a}\right)^2 = 3^2$$

$$a^2 + 1 + 1 + \frac{1}{a^2} = 9$$

$$a^2 + 2 + \frac{1}{a^2} = 9$$

$$a^2 + \frac{1}{a^2} = \mathbf{7}.$$

6. Since r is a solution to $x^2 + 11x - 19 = 0$, we know that $r^2 + 11r - 19 = 0$. We are looking for the value of $(r + 5)(r + 6) = r^2 + 11r + 30$. Taking $r^2 + 11r - 19 = 0$ and adding 49 to each side, we get $r^2 + 11r + 30 = \mathbf{49}$.

7. Using properties of exponents to rearrange the equation, we get

$$\left(\frac{1}{4}\right)^{2x+8} = (16)^{2x+5}$$

$$\frac{1}{(4)^{2x+8}} = (4^2)^{2x+5}$$

$$1 = (4)^{4x+10}(4)^{2x+8}$$

$$1 = (4)^{6x+18}$$

In order for the expression to work, $6x + 18$ must equal 0. The value of x is $\mathbf{-3}$.

8. First, we rewrite the equation $4^x = 33 \cdot 2^{x-1} - 8$ as $4^x - 33 \cdot 2^{x-1} + 8 = 0$. Since $4 = 2^2$, it follows that $4^x = (2^2)^x = (2^x)^2$. Also, since $2^{x-1} = 2^x \cdot 2^{-1} = 2^x \cdot (1/2)$, we can write $(2^x)^2 - 33 \cdot 2^x \cdot (1/2) + 8 = 0 \rightarrow (2^x)^2 - 2^x \cdot (33/2) + 8 = 0$. Now we can let $y = 2^x$, and rewrite the equation as $y^2 - (33/2)y + 8 = 0$. To eliminate the fraction, we can multiply each side of the equation by 2, to get $2y^2 - 33y + 16 = 0$. Factoring the trinomial, we get $(2y - 1)(y - 16) = 0$. So, $2y - 1 = 0 \rightarrow 2y = 1 \rightarrow y = 1/2$, and $y - 16 = 0 \rightarrow y = 16$ are solutions to this quadratic equation. To solve the original equation, we substitute 1/2 and 16 for y in the equation $y = 2^x$. We have $1/2 = 2^x \rightarrow 2^{-1} = 2^x \rightarrow x = \mathbf{-1}$, and $16 = 2^x \rightarrow 2^4 = 2^x \rightarrow x = \mathbf{4}$.