Warm-Up!

1. The distance from Goteborg to Jonkiping measured on the map is 88 cm, and every 1 cm on the map is equivalent to 15 km. Therefore, the actual distance is $88 \times 15 = 1320$ km.

2. If the profits are divided in a ratio 2:3:3:5, then the partner receiving the largest profit would receive $\frac{5}{2 + 3 + 3 + 5} = \frac{5}{13}$ of the total profit or $\frac{5}{13} \times 26,000 = 10,000$.

3. Since adding 700 gallons brought the tank from 26% full to 40% full, 700 gallons is equivalent to $40 - 26 = 14\%$ of the tank's capacity. If the tank has capacity of $x$ gallons we have $0.14x = 700$ and $x = 700/0.14 = 5,000$ gallons.

4. The ratio of round tables to rectangular tables is 2:1; therefore, $\frac{2}{3}$ of the total number of tables are round and $\frac{1}{3}$ are rectangular. Each round table seats 8, and each rectangular table seats 12, with the total number of seats being 336. We can set up the following equation with $t$ representing the total number of tables:

\[
\frac{2}{3} \times 8 \times t + \frac{1}{3} \times 12 \times t = 336.
\]

Solving for $t$, we get $\frac{28}{3} \times t = 336$ and $t = 336 \times \frac{3}{28} = 36$ tables.

The Problems are solved in the MATHCOUNTS Mini video.

Follow-up Problems

5. The ratio at the start is $4/7 = r/g$, where $r$ is the number of red marbles and $g$ is the number of green marbles in the bag initially. Cross multiplying, we get $7r = 4g$. After adding 90 red marbles and 36 green marbles we are told that the probability of randomly drawing a red marble is $1/2 = 50\%$. That means the number of red marbles equals the number of green marbles in the bag. So, it follows that $r + 90 = g + 36$ and $g - r = 54$. Solving the system of equations we get $4g - 4r = 54(4) \rightarrow 7r - 4r = 216 \rightarrow 3r = 216 \rightarrow r = 72$ and $g = 72 + 54 = 126$. The problem asks for the total number of marbles after the 126 new marbles have been added, this will be $72 + 126 + 126 = 324$ marbles.

6. Using $J$, $B$ and $M$ for John's, Bill's and Mary's allowances, respectively, we can set up the following ratios: $J/B = 3/7$ and $M/J = 2/5$. The ratio of Mary's allowance to Bill's allowance is $M/B = J/B \times M/J = 3/7 \times 5/2 = 15/14$.

7. Let's use the variables $P$ and $Q$ to represent the two distinct positive numbers. The problem states that $P^2 - Q^2 = 2(P - Q)^2$. This equation can be factored to $(P - Q)(P + Q) = 2(P - Q)(P - Q)$. Dividing both sides by $P - Q$, we get $P + Q = 2(P - Q)$ or $P + Q = 2P - 2Q$. Combining like terms, we get $3Q = P$. Since we are looking for the ratio of the smaller number to the larger number, we arrive at $Q/P = 1/3$.

8. We know that the surface area of JZ114 will be $10^2$ times larger than that of our sun since we are told that its radius is 10 times larger than the sun. Since the formula for energy given off is $\sigma AT^4$, and $\sigma$ is a constant, the ratio of the energy from JZ114 to our sun will be $10^2 A(\frac{1}{2} T)^4/A T^4 = 10^2 \times (1/2)^4 = 100 \times 1/16 = 25/4$.  

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