MATHCOUNTS® Minis January 2016 Activity Solutions

Warm-Up!

1. The distance from Goteborg to Jonkiping measured on the map is 88 cm, and every 1 cm on the map is equivalent to 15 km. Therefore, the actual distance is 88 × 15 = **1320** km.

2. If the profits are divided in a ratio 2:3:3:5, then the partner receving the largest profit would receive 5/(2 + 3 + 3 + 5) = 5/13 of the total profit or 5/13 × $26,000 = $**10,000**.

3. Since adding 700 gallons brought the tank from 26% full to 40% full, 700 gallons is equivalent to 40 − 26 = 14% of the tank's capacity. If the tank has capacity of x gallons we have $0.14x = 700$ and $x = 700/0.14$ = **5,000** gallons.

4. The ratio of round tables to rectangular tables is 2:1; therefore, 2/3 of the total number of tables are round and 1/3 are rectangular. Each round table seats 8, and each rectangular table seats 12, with the total number of seats being 336. We can set up the following equation with t representing the total number of tables: $2/3 \times 8 \times t + 1/3 \times 12 \times t = 336$. Solving for t, we get $28/3 \times t = 336$ and $t = 336 \times 3/28$ = **36** tables.

The Problems are solved in the MATHCOUNTS® Minis video.

Follow-up Problems

5. The ratio at the start is $4/7 = r/g$, where r is the number of red marbles and g is the number of green marbles in the bag initially. Cross multiplying, we get $7r = 4g$. After adding 90 red marbles and 36 green marbles we are told that the probability of randomly drawing a red marble is 1/2 = 50%. That means the number of red marbles equals the number of green marbles in the bag. So, it follows that $r + 90 = g + 36$ and $g - r = 54$. Solving the system of equations we get $4g - 4r = 54(4) \rightarrow 7r - 4r = 216 \rightarrow 3r = 216 \rightarrow r = 72$ and $g = 72 + 54 = 126$. The problem asks for the total number of marbles after the 126 new marbles have been added, this will be 72 + 126 + 126 = **324** marbles.

6. Using J, B and M for John's, Bill's and Mary's allowances, respectively, we can set up the following ratios: $J/B = 3/7$ and $M/J = 2/5$. The ratio of Mary's allowance to Bill's allowance is $M/B = J/B \times M/J = 3/7 \times 5/2$ = **15/14**.

7. Let's use the variables P and Q to represent the two distinct positive numbers. The problem states that $P^2 - Q^2 = 2(P - Q)^2$. This equation can be factored to $(P - Q)(P + Q) = 2(P - Q)(P - Q)$. Dividing both sides by $P - Q$, we get $P + Q = 2(P - Q)$ or $P + Q = 2P - 2Q$. Combining like terms, we get $3Q = P$. Since we are looking for the ratio of the smaller number to the larger number, we arrive at Q/P = **1/3**.

8. We know that the surface area of JZ114 will be 10^2 times larger than that of our sun since we are told that its radius is 10 times larger than the sun. Since the formula for energy given off is σAT^4, and σ is a constant, the ratio of the energy from JZ114 to our sun will be $10^2 A(\frac{1}{2}T)^4 / AT^4 = 10^2 \times (1/2)^4 = 100 \times 1/16$ = **25/4**.