

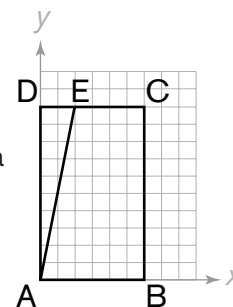
Warm-Up!

1. Since we are told that ABCD is a parallelogram, we know that opposite sides must be parallel. Side AB is parallel to side DC. It follows, then, that side AD must be parallel to side BC. The slope of side BC is $(1 - 0)/(2 - 4) = -1/2$. The point that is located one unit down and two units right from A(0, 1) is D(2, 0). The sum of the coordinates of D is $2 + 0 = 2$.

2. The triangle with vertices (4, -1), (4, 5) and (10, 3) has base length $5 - (-1) = 6$ and height $10 - 4 = 6$. Its area, then, is $(1/2) \times 6 \times 6 = 18$ units².

3. The slope of a line perpendicular to line k will have a slope that is the opposite reciprocal of that of line k . Using the slope formula for line k , we get $(-5 - 8)/(2 + 1) = -13/3$. Therefore, the slope of a line perpendicular to line k is **3/13**.

4. A segment drawn connecting A with E, as shown, creates a right triangle ADE and trapezoid ABCE. Triangle ADE has legs of length 10 units and 2 units, making its area $(1/2) \times 10 \times 2 = 10$ units². The area of trapezoid ABCE is difference between the area rectangle ABCD and the area of triangle ADE. Rectangle ABCD has area $10 \times 6 = 60$ units². So, the area of trapezoid ABCE is $60 - 10 = 50$ units². The ratio of the area of triangle ADE to the area of trapezoid ABCE, then, is $10/50 = 1/5$.

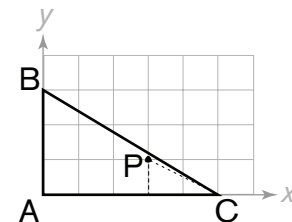


The Problems are solved in the **MATHCOUNTS**® *Mini*® video.

Follow-up Problems

5. We know that the distances from P to vertices A and B are $\sqrt{10}$ units and $\sqrt{13}$ units, respectively. If we let the coordinates of point P be (x, y) , then $\sqrt{[(x - 0)^2 + (y - 0)^2]} = \sqrt{10} \rightarrow x^2 + y^2 = 10$ and $\sqrt{[(x - 0)^2 + (y - 3)^2]} = \sqrt{13} \rightarrow x^2 + (y - 3)^2 = 13 \rightarrow x^2 + y^2 - 6y + 9 = 13 \rightarrow x^2 + y^2 - 6y = 4$. Subtracting the equations $x^2 + y^2 = 10$ and $x^2 + y^2 - 6y = 4$, we get $6y = 6 \rightarrow y = 1$.

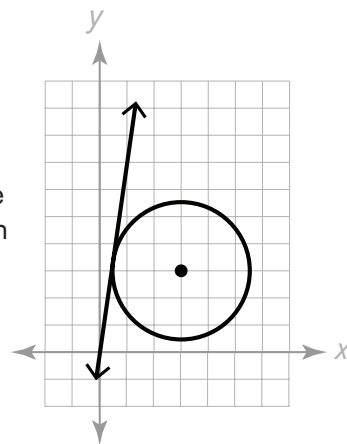
Substituting for y in the equation $x^2 + y^2 = 10$ yields $x^2 + 1 = 10 \rightarrow x^2 = 9 \rightarrow x = 3$. Now we know that the coordinates of P are (3, 1). Notice that segment PC is the hypotenuse of a triangle with leg lengths 1 and 2. Using the Pythagorean Theorem, we have $1^2 + 2^2 = PC^2 \rightarrow 5 = PC^2 \rightarrow PC = \sqrt{5}$. So the distance from P to vertex C is **$\sqrt{5}$** units.



6. For any two points A and B on a circle, the perpendicular bisector of segment AB passes through the center of the circle, (h, k) . The line that bisects and is perpendicular to the segment joining (3, 0) and (9, 0) is the vertical line $x = 6$. So the center of the circle has the form $(6, k)$. Next, note that the midpoint of the segment with endpoints $(-1, 2)$ and $(3, 0)$ is $((-1 + 3)/2, ((2 + 0)/2)) = (1, 1)$, and the slope of the segment is $(0 - 2)/(3 - (-1)) = -1/2$. The perpendicular bisector of this segment, then, passes through $(1, 1)$ and has slope 2. Substituting these values in the point-slope form, we see that this line is given by the equation $y - 1 = 2(x - 1) \rightarrow y - 1 = 2x - 2 \rightarrow y = 2x - 1$. Since this line passes through center $(6, k)$, we can substitute to determine the value of k . We get $y = 2(6) - 1 = 12 - 1 = 11$. Therefore, the circle has center $(h, k) = (6, 11)$, and $h + k = 6 + 11 = 17$.

7. Any point that is equidistant from $(3, -1)$ and $(7, -9)$ will be on the perpendicular bisector of the segment joining these two points. So if we find an equation for that line we can determine where it will intersect the line given by $x + y = 3$. The midpoint of the segment from $(3, -1)$ to $(7, -9)$ is $((3 + 7)/2, (-1 - 9)/2) = (5, -5)$. The slope of the segment is $(-1 + 9)/(3 - 7) = 8/(-4) = -2$. It follows, then, that the slope of the perpendicular bisector is $1/2$. Substituting for m , x and y in the point slope form, we have $y + 5 = (1/2)(x - 5) \rightarrow y + 5 = (1/2)x - 5/2 \rightarrow -(1/2)x + y = -15/2$. Multiplying by 2 to eliminate the fractions, we have $-x + 2y = -15$. To determine the intersection of the two lines given by $x + y = 3$ and $-x + 2y = -15$, let's add the two equations as follows: $(x + y) + (-x + 2y) = 3 - 15 \rightarrow 3y = -12 \rightarrow y = -4$. Substituting this value into $x + y = 3$, we get $x - 4 = 3 \rightarrow x = 7$. Thus, the point on $x + y = 3$ that is equidistant from $(3, -1)$ and $(7, -9)$ has coordinates $(7, -4)$.

8. The graph of $(x - 3)^2 + (y - 3)^2 = 6$ is a circle with center $(3, 3)$ and radius $\sqrt{6}$. If (x, y) satisfies this equation, then the point (x, y) is on the circle. Furthermore, if $y/x = k$, then the point (x, y) is on the line through the origin with slope k given by the equation $y = kx$. We are looking for the point on the circle such that the line through the point and the origin has the greatest possible slope. This occurs when the line passes through the origin and is tangent to the circle. Any line with greater slope will not intersect the circle, since we must rotate the tangent line counter-clockwise about the origin (away from the circle) to produce a line of greater slope. Any other line that intersects the circle is the result of rotating the tangent line about the origin towards the center of the circle, reducing the slope of the line.



Since the point we're looking for is the only point that satisfies both $y = kx$ and $(x - 3)^2 + (y - 3)^2 = 6$ for the desired value of k , we are looking for k such that the system of equations has exactly one solution. Substituting $y = kx$ in the equation gives us $(x - 3)^2 + (kx - 3)^2 = 6 \rightarrow x^2 - 6x + 9 + k^2x^2 - 6kx + 9 = 6 \rightarrow (k^2 + 1)x^2 + (-6 - 6k)x + 12 = 0$. This quadratic equation has only one solution for x if its discriminant equals 0. The discriminant of a quadratic equation of the form $ax^2 + bx + c = 0$ is $b^2 - 4ac$. In this case, the discriminant is $(-6 - 6k)^2 - 4(k^2 + 1)(12) = 36 + 72k + 36k^2 - 48k^2 - 48 = -12k^2 + 72k - 12 = -12(k^2 - 6k + 1)$. We can solve the quadratic equation $-12(k^2 - 6k + 1) = 0 \rightarrow k^2 - 6k + 1 = 0$, with $a = 1$, $b = -6$ and $c = 1$, using the quadratic formula as follows: $k = [-b \pm \sqrt{(b^2 - 4ac)}]/2a = [6 \pm \sqrt{((-6)^2 - 4(1)(1))}]/(2)(1) = [6 \pm \sqrt{(36 - 4)}]/2 = (6 \pm \sqrt{32})/2 = (6 \pm 4\sqrt{2})/2 = 3 \pm 2\sqrt{2}$. The two values of k are the slopes of the two tangents from the origin to the circle. We are interested in the larger slope, which is $3 + 2\sqrt{2}$.