MATHCOUNTS® Minis

February 2016 Activity Solutions

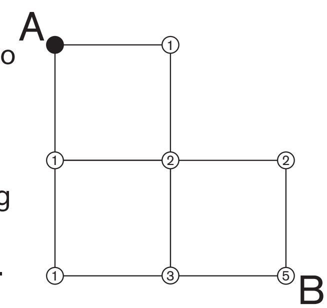
Warm-Up!

1. We are asked to determine how many permutations of the five letters in the word HOOPS have the two Os adjacent. If we consider the two Os a single object, then we need to determine the number of permutations of four objects. There are 4! = 4 x 3 x 2 x 1 = **24** such permutations.

2. If a four-digit number has exactly one 0 it must be in the units, tens or thousands place. In any of these three cases the remaining three digits can be 1, 2, 3, 4, 5,6,7,8 or 9. That means that there are (9 x 9 x 9 x 1) x 3 = 729 x 3 = **2187** such four-digit numbers.

3. There are three dice, and six outcomes when rolling each die. If we are interested in rolling three different numbers then there are 6 × 5 × 4 = 120 ways this can occur. The total number of outcomes when rolling three dice is 6 × 6 × 6 = 216 outcomes. Therefore, the probability of rolling three different numbers is 120/216 = **5/9**.

We can also consider the probability of two independent events that occur once the first die is rolled. When the second die is rolled, the probability of getting a different number than the first roll is 5/6. And when the third die is rolled the probability of getting a different number than the first two rolls is 4/6 = 2/3. Therefore, the probability of getting three different numbers is (5/6) × (2/3) = 10/18 = **5/9**.



4. One way to solve this type of problem is to indicate the number of ways to get to each intersection on the grid, as shown. The ① at the location to the right of A indicates that there is only one path from A to that location. The same is true for the location just below A. The ② to the immediate right of that location indicates that there are two paths from A to it, one approaching from above and the other from the left. There are also two paths from A to the point to the immediate right of this location as indicated by the ② there. The ① at the point in the bottom left corner implies that there is only one path to it from A. To get to the location labeled ③ in the manner described, the ladybug must pass through the point immediately above it, to which there are two paths from A, or it must approach from the point to the immediate left, to which there is one path from A. Thus, there are 2 + 1 = 3 paths to this location. Finally, the ladybug can only reach B by going through the point above B, to which there are two paths, or the point to the left of B, to which there are three paths. It follows, then, that there are 2 + 3 = **5** paths the ladybug can take to walk from A to B.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. Let's use the Fundamental Counting Principle to count the number of four-digit numbers. Since the first digit can not be zero, there are 9 choices for the first digit. Since the second digit must be different from the first (and can be zero), there are also 9 choices for the second digit. Similiarly, there are 9 choices for the third digit and 9 choices for the fourth digit. That's a total of 9 × 9 × 9 × 9 = **6561** four-digit positive integers.

6. Let's think of the possible choices for each of the digits in a 7-digit palindrome. Since the first digit can not be zero, there are 9 possible choices for the first digit. The second, third and fourth digits can each be any number since we were not told the digits have to be distinct. So, there are 10 possible choices each for the second, third and fourth digits. The fifth digit must be the same as the third digit; the sixth must be the same as the second; and the seventh must be the same as the first. Thus, there is only one choice for each of the fifth, sixth and seventh digits. Therefore, there are are $9 \times 10 \times 10 \times 10 \times 1 \times 1 \times 1 =$ **9000** possible 7-digit palindromes.

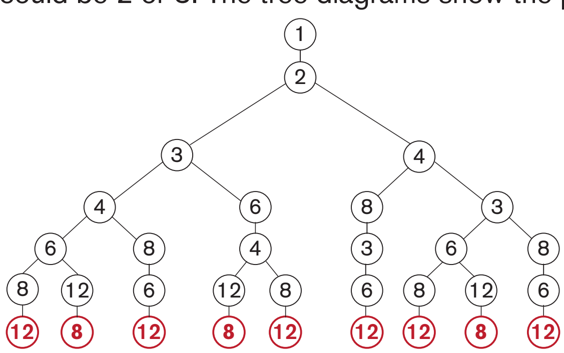
Alternatively, for every 4-digit number (ABCD), we can create a unique 7-digit palindrome (A,BCD,CBA), and the first 4 digits of every 7-digit palindrome create a unique 4-digit number. There are 9000 4-digit numbers from 1000 through 9999, and each of these can be made into one of the **9000** 7-digit palindromes.

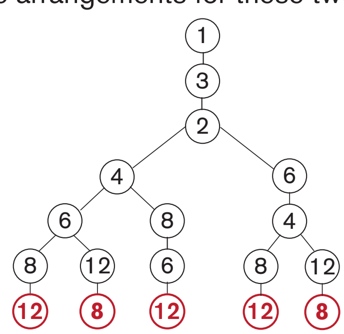
7. Since Ben (B) insists on sitting next to Amy (A), we'll consider them as a single entity. Similarly, we'll consider the pairing of Carol (C) and Dave (D) as a single entity. So two seats will be occuppied by AB or BA, and two seats will be occupied by CD or DC. We have the following combinations of these two pairings:

AB	CD	___	___	___
AB	DC	___	___	___
BA	CD	___	___	___
BA	DC	___	___	___

That leaves three seats to fill. Let E, F and G represent the three other students, and consider this possible seating arrangement: AB | CD | E | F | G. For these 5 entities (AB, CD, E, F, G), there are 5! = 120 different seating arrangements. In fact, there are 120 possible seating arrangements for each of the combinations of the pairings listed above. That means there are $120 \times 4 =$ **480** different ways in which to seat these seven students.

8. We will use tree diagrams to aid in counting the arrangements. Any arrangement that meets the given criteria must begin with 1 since it is a divisor of every number in the set. After 1, the second number could be 2 or 3. The tree diagrams show the possible arrangements for these two cases.





There are 9 ways to order the numbers beginning with 1, 2, ..., and there are 5 ways to order them beginning with 1, 3,... There are a total of 9 + 5 = **14** ways this set of numbers can be ordered.