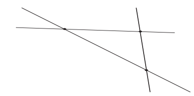
February 2014 Activity Solutions

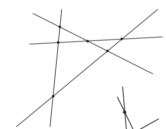
Warm-Up!

1. We are asked to determine how many permutations of the five letters in the word HOOPS have the two Os adjacent. If we consider the two Os a single object, then we need to determine the number of permutations of four objects. There are 4! = 4 x 3 x 2 x 1 = **24** such permutations.

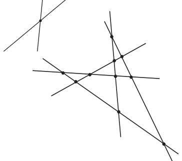
2. If a four-digit number has exactly one 0 it must be in the units, tens or thousands place. In any of these three cases the remaining three digits can be 1, 2, 3, 4, 5,6,7,8 or 9. That means that there are (9 x 9 x 9 x 1) x 3 = 729 x 3 = **2187** such four-digit numbers.



3. The maximum number of points of intersection when 3 lines are drawn in a plane, as shown, is **3** points.

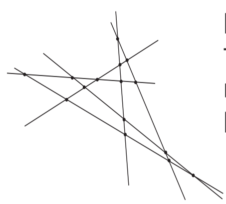


4. The maximum number of points of intersection when 4 lines are drawn in a plane, as shown, is **6** points.



5. The maximum number of points of intersection when 5 lines are drawn in a plane, as shown, is **10** points.

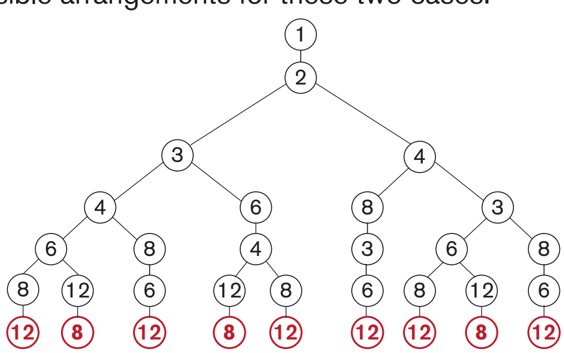
6. Based on the previous problems, it appears that the number of points of intersection when 3, 4 and 5 lines are drawn in a plane form the sequence 3, 6, 10, ..., where the difference between each pair of successive terms is one more than the difference between the previous two terms. Observe that 3 + 3 = 6, and 6 + 4 = 10. So, when 6 lines are drawn in a plane, the number of points of intersection to be 10 + 5 = 15. The figure shown confirms this. Continuing the sequence, we have 3, 6, 10, 15, 21, 28, 35, 45, 56, ... If follows, then, that the maximum number of points of intersection when 10 lines are drawn in a plane would be the 8th term in the sequence, or **45** points.

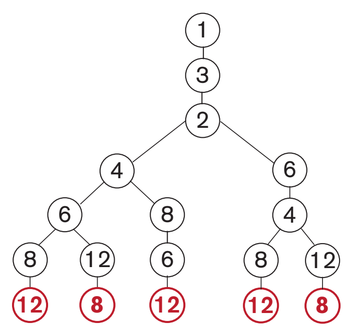


The Problems are solved in the video.

Follow-up Problems

7. Using the technique employed in the video, we will use tree diagrams to aid in counting the arrangements. Any arrangement that meets the given criteria must begin with 1 since it is a divisor of every number in the set. After 1, the second number could be 2 or 3. The tree diagrams show the possible arrangements for these two cases.





There are 9 ways to order the numbers beginning with 1, 2, ..., and there are 5 ways to order them beginning with 1, 3,... There are a total of 9 + 5 = **14** ways this set of numbers can be ordered.

8. Since Ben (B) insists on sitting next to Amy (A), we'll consider them as a single entity. Similarly, we'll consider the pairing of Carol (C) and Dave (D) as a single entity. So two seats will be occupied by AB or BA, and two seats will be occupied by CD or DC. We have the following combinations of these two pairings:

AB | CD | ___ | ___ | ___

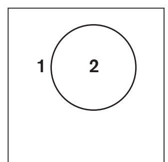
AB | DC | ___ | ___ | ___

BA | CD | ___ | ___ | ___

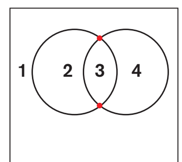
BA | DC | ___ | ___ | ___

That leaves three seats to fill. Let E, F and G represent the three other students, and consider this possible seating arrangement: AB | CD | E | F | G. For these 5 entities (AB, CD, E, F, G), there are 5! = 120 different seating arrangements. In fact, there are 120 possible seating arrangements for each of the combinations of the pairings listed above. That means there are 120 × 4 = **480** different ways in which to seat these seven students.

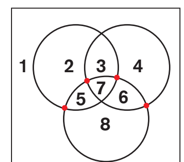
9. This is similar to the pizza problem solved in the video. We know that one circle divides the plane into 2 regions, as shown. We could draw another circle in the plane that does not intersect the original circle, but that only creates 1 additional region (inside the new circle). We can maximize the number of regions if the circles intersect. As shown, two circles can divide the plane, at most, into 4 regions. Two new regions are formed where one circle *cuts* the other circle. In order to maximize the number of regions when adding a third circle, again we'll draw the new circle and maximize the number of *cuts*. Now, let's review. When a second circle intersects the first circle, it *cuts* the first circle in two locations, and 2 new regions are formed. When a third circle intersects each of the first two circles, there are four *cuts* (two per circle), and 4 new regions are formed. When a fourth circle is drawn so that it intersects each of the three existing circles, making a total of 6 new *cuts*, we would expect 6 new regions to be formed. And when a fifth circle is drawn so that it intersects each of the four existing circles, making a total of eight new *cuts*, we would expect 8 new regions to be formed. Thus, the maximum number of regions formed by 5 circles in a plane is 2 + 2+ 4 + 6 + 8 = **22** regions. This is confirmed by the figures below.



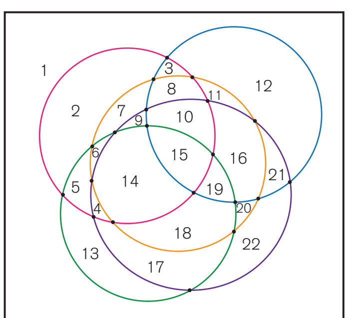
1 circle



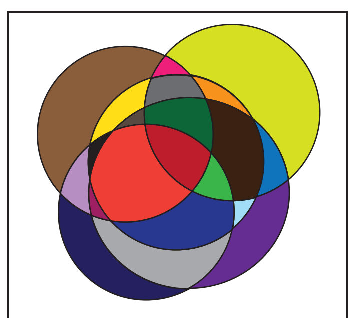
2 circles



3 circles



5 circles


5 circles