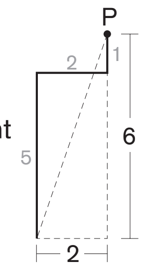
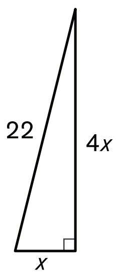
MATHCOUNTS® Minis December 2017 Activity Solutions

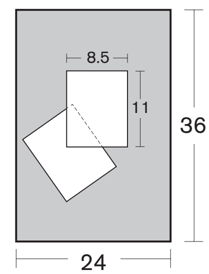
Warm-Up!



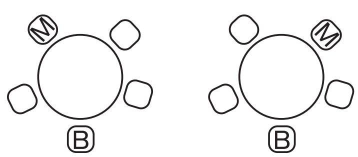
1. As the figure shows, Samantha walks a total of 5 + 1 = 6 mi north and 2 mi east. The distance we are looking for, from her starting point to P, is the length of the hypotenuse of a right triangle with legs of length 6 and 2. Using the Pythagorean Theorem, we have $d^2 = 6^2 + 2^2 \rightarrow d^2 = 36 + 4 \rightarrow d^2 = 40 \rightarrow d = \sqrt{40} \rightarrow d =$ **$2\sqrt{10}$** miles.



2. Assuming that the ground is perpendicular to the vertical structure, we can draw a right triangle, as shown, with legs of x and $4x$ feet and a hypotenuse of 22 feet. We can then use the Pythagorean Theorem and solve for x as follows: $x^2 + (4x)^2 = 22^2 \rightarrow 1x^2 + 16x^2 = 484 \rightarrow 17x^2 = 484 \rightarrow x^2 = 484/17 \rightarrow x = \sqrt{(484/17)}$. The value of x tells us how far the base of the ladder can be from the wall. We want to know how high the ladder can safely reach up the vertical structure, which is $4x$, or about $4 \times \sqrt{(484/17)} \approx$ **21.3** feet.



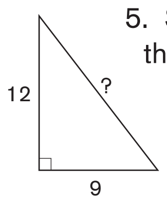
3. Since 1 ft = 12 inches, the tabletop has a total area of 24 × 36 = 864 in^2. Each sheet of paper has an area of 8.5 × 11 = 93.5 in^2. If there was no overlap between the two sheets of paper, there would be a total of 864 − (93.5 × 2) = 864 − 187 = 677 in^2 not covered by the sheets. We are told that the total uncovered area is 700 in^2, so the area of overlap must be 700 − 677 = **23** in^2.



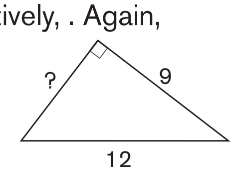
4. Since Melissa does not sit next to Bill, there are only two possible places for her to sit once Bill is seated as shown. In both cases there are 3! = 6 ways for the three remaining friends to be seated. That's a total of 2 × 6 = **12** distinct seating arrangements.

The Problems are solved in the MATHCOUNTS® Mini video.

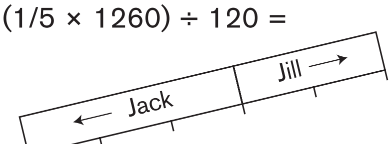
Follow-up Problems

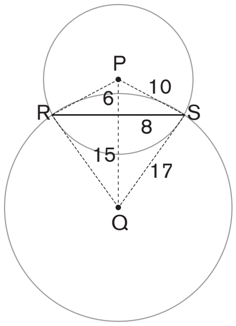


5. Suppose, as shown on the left, that the right triangle has legs of lengths 9 and 12. Based on the Pythagorean Theorem, the hypotenuse has length $\sqrt{(9^2 + 12^2)} = \sqrt{(81 + 144)} = \sqrt{225} = 15$. Since the hypotenuse is the side of greatest length in a right triangle, suppose, as shown on the right, that 9 and 12 are the lengths of a leg and the hypotenuse, respectively, . Again, based on the Pythagorean Theorem, the other leg has length $\sqrt{(12^2 - 9^2)} = \sqrt{(144 - 81)} = \sqrt{63} = 3\sqrt{7}$. The product of the these two numerical values is $15 \times 3\sqrt{7} =$ **$45\sqrt{7}$.**



6. As the figure shows, Jack travels down 3/5 of the hill in the same 2 minutes = 120 seconds that Jill travels up 2/5 of the hill. Using the formula rate = distance ÷ time, we see that the difference in Jack's and Jill's speeds is [(3/5 × 1260) ÷ 120] − [(2/5 × 1260) ÷ 120] = (1/5 × 1260) ÷ 120 = 252/120 = **2.1** ft/s.





7. From the information given, we can construct the figure with intersecting circles P and Q of radius 10 and 17 feet, respectively. As shown, the common chord RS is bisected by segment PQ. When we draw the radii from center P to R and S, we create congruent right triangles whose side lengths form the 6-8-10 Pythagorean Triple. Similarly, when we draw the radii from center Q to R and S, we create two other congruent right triangles whose side lengths form the 8-15-17 Pythagorean Triple. Thus, the length of segment PQ is 6 + 15 = **21** feet.

8. Let *a* represent the length of the shortest piece. The table shows the four ways to cut a 36-inch piece of rope into three pieces so that one piece is five inches longer than another, and one piece is twice as long as another.

	SHORT	MEDIUM	LONG
I	a	$a + 5$	$2a$
II	a	$2a$	$2a + 5$
III	a	$a + 5$	$2(a + 5)$
IV	a	$2a - 5$	$2a$

We can solve the following equations to determine the value of *a* and the length of the longest piece in each case.

For case I, $a + a + 5 + 2a = 36 \rightarrow 4a + 5 = 36 \rightarrow 4a = 31 \rightarrow a = 31/4$ inches, and the longest piece measures $2a = 2(31/4) = \underline{31/2}$ inches.

For case II, $a + 2a + 2a + 5 = 36 \rightarrow 5a + 5 = 36 \rightarrow 5a = 31 \rightarrow a = 31/5$ inches, and the longest piece measures $2a + 5 = 2(31/5) + 5 = \underline{87/5}$ inches.

For case III, $a + a + 5 + 2(a + 5) = 36 \rightarrow 4a + 15 = 36 \rightarrow 4a = 21 \rightarrow a = 21/4$ inches, and the longest piece measures $2(a + 5) = 2(21/4 + 5) = 82/4 = \underline{41/2}$ inches.

For case IV, $a + 2a - 5 + 2a = 36 \rightarrow 5a - 5 = 36 \rightarrow 5a = 41 \rightarrow a = 41/5$ inches, and the longest piece measures $2a = 2(41/5) = \underline{82/5}$ inches.

We must also consider the case in which the piece that is twice the length of another ALSO is five inches longer than that piece. In other words, when $2a = a + 5 \rightarrow a = 5$ inches. In this case, the longest piece has length $36 - (5 + 10) = \underline{21}$ inches. The sum of all the possible lengths of the longest piece, then, is $31/2 + 87/5 + 41/2 + 82/5 + 21 = (155 + 174 + 205 + 164 + 210)/10 = 908/10 =$ **90.8** inches.