MATHCOUNTS® Minis December 2016 Activity Solutions

Warm-Up!

1. One line divides the plane into two regions (Fig. 1). A second line that intersects the first line divides each of those regions into two regions, creating four regions (Fig. 2). A third line that intersects the first two lines divides three of these regions into two regions each (Fig. 3). That brings the total number of regions determined by three lines in a plane to **7** regions, as shown.

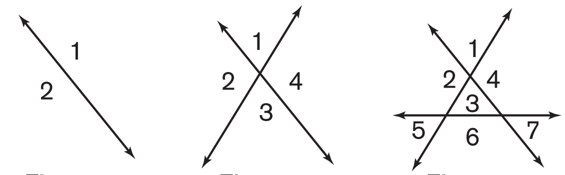

Figure 1 **Figure 2** **Figure 3**

2. The prime factorization of 182 is 2 × 7 × 13, so it has 2 × 2 × 2 = 8 factors. They are 1, 2 ,7, 13, 14, 26, 91 and 182. The pair of factors closest to one another in value, 13 and 14, yields the least possible sum, which is 13 + 14 = **27**.

3. There are two ways to get a sum of 65 from two perfect squares: $8^2 + 1^2 = 64 + 1 = 65$ and $7^2 + 4^2 = 49 + 16 = 65$. Since $x > y$, the two values for $x - y$ are 64 − 1 = 63 and 49 − 16 = 33. Therefore, the lowest possible value of $x - y$ is **33**.

4. From the information given, we can write the equation $(4 + 20 + x)/3 = (y + 16)/2$. Cross-multiplying, we get $2 \times (24 + x) = 3 \times (y + 16) \rightarrow 48 + 2x = 3y + 48 \rightarrow 2x = 3y$. Since we want the smallest possible value of the positive integers x and y, we can make $x = 3$ and $y = 2$. The value of $x + y$, then, is 3 + 2 = **5**.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. If $2015 + a = b$, then $a = b - 2015$. Since a and b are both positive integers, we start subtracting 2015 from each of the possible b palindromes, beginning with 2112, which is the smallest palindrome greater than 2015. As the table shows, the first difference we obtain that also is a palindrome is 2772 − 2015 = **757**.

b	2112	2222	2332	2442	2552	2662	**2772**
a	97	207	317	427	537	647	**757**

6. Let n represent the largest of the six numbers arranged in ascending order. To maximize n, the other five numbers must be as small as possible. The smallest possible value for the first two numbers is 1. Since the median, in this case 8, of the ordered list is the average of the 3rd and 4th numbers, and since the 5th number can be no less than the 4th number, it follows that 8 is the smallest possible value for these three numbers. Finally, we know that the mean of 1, 1, 8, 8, 8 and n is 6, so it follows that $(1 \times 2 + 8 \times 3 + n) = 36 \rightarrow 26 + n = 36 \rightarrow n =$ **10**.

7. We want the smallest value for the required $a + b + c$, so let's try splitting up a small square number into a and b and see how that works out. Suppose we try $a + b = 1 + 8 = 9$. Then the two other squares will be $8 - 1 = 7$ apart. The squares 9 and 16 are 7 apart, but that makes $c = 8$, which is a repeated number. If we try $a + b = 2 + 7 = 9$, we need two squares that are $7 - 2 = 5$ apart. Those squares would be 4 and 9, but that makes $c = 2$, which is again a repeat. Since starting with 9 doesn't work, we move on to 16. If we try $a + b = 1 + 15 = 16$, the other two squares would be $15 - 1 = 14$ apart, which doesn't happen. If we try $a + b = 2 + 14 = 16$, the other two squares would be $14 - 2 = 12$ apart. They would be 4 and 16, which lead to another repeat. In some cases, we would need a negative number. For example, if $a + b = 4 + 12 = 16$, then the other two squares would be 8 apart. They would be 1 and 9, but then c would have to be -3. Continuing the search, we eventually come to the solution $a + b = 6 + 19 = 25$. The other two squares must be $19 - 6 = 13$ apart. They would be 36 and 49, so c would have to be 30. Our solution is $a = 6$, $b = 19$ and $c = 30$. The three squares are $6 + 19 = 25$, $6 + 30 = 36$ and $19 + 30 = 49$. The sum $a + b + c$ is $6 + 19 + 30 =$ **55**.

8. First, let's consider three players. If they are positioned so that no two of them stand the same distance apart, then we have a scalene triangle. The longest side of a triangle is opposite the greatest angle, so the player standing at the vertex of the angle with the greatest degree measure can get hit by both of the other players. Whatever the degree measure of the greatest angle, it must be greater than 60 degrees. We now can imagine fitting several triangles together so that one player stands at the vertex of the angle with the greatest measure in each of these non-overlapping triangles. Since $360 \div 60 = 6$, we can divide 360 degrees into, at most, 5 angles of measure greater than 60 degrees. Thus, the maximum number of times a player can get hit is **5** times.