MATHCOUNTS® Minis September 2019 Activity Solutions

Warm-Up!

1. Simplifying, we get $\frac{5!\cdot 2!}{3!} = \frac{5\cdot 4\cdot \cancel{3!}\cdot 2!}{\cancel{3!}} = 5\cdot 4\cdot 2\cdot 1 = \mathbf{40}$.

2. Simplifying, we get $\frac{18!}{16!} = \frac{18\cdot 17\cdot \cancel{16!}}{\cancel{16!}} = 18\cdot 17 = \mathbf{306}$.

3. Simplifying, we get $n! = \frac{10!}{7!\cdot 3!} = \frac{10\cdot 9\cdot 8\cdot \cancel{7!}}{\cancel{7!}\cdot 3!} = \frac{10\cdot \cancel{9}^{3}\cdot \cancel{8}^{4}}{\cancel{3}\cdot \cancel{2}\cdot 1} = 10\cdot 3\cdot 4 = 2\cdot 5\cdot 3\cdot 4 = 5!$. So, n = **5**.

4. Simplifying, we get $\frac{6!+5!}{5!} = \frac{6\cdot \cancel{5!}+\cancel{5!}}{\cancel{5!}} = 6+1 = \mathbf{7}$.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. Simplifying, we get $\frac{8!+9!}{7!+8!} = \frac{8\cdot \cancel{7!}+9\cdot 8\cdot \cancel{7!}}{\cancel{7!}+8\cdot \cancel{7!}} = \frac{8+9\cdot 8}{1+8} = \frac{8+72}{9} = \mathbf{\frac{80}{9}}$.

6. Since 7! = 7 · 6 · 5 · 4 · 3 · 2 · 1 = 7 · (2 · 3) · 5 · (2 · 2) · 3 · 2 · 1, we see that the greatest perfect square factor of 7! is $(2^2 \cdot 3)^2 = 12^2$ = **144**.

7. Since 10 = 2 · 5, let's see how many 2s and 5s are factors of 25! = 25 · 24 · 23 · ··· · 3 · 2 · 1. There are 5 multiples of 5 in this expanded representation of 25!, namely 5, 10, 15, 20 and 25. There is one factor of 5 in each of 5, 10 15 and 20, and there are two factors of 5 in 25, for a total of six 5s that are factors of 25!. The 12 even numbers in our expanded representation of 25! all are multiples of 2, so we know that the number of 2s that are factors is at least 12. Pairing each 5 factor with a 2 factor, we see that the greatest power of 10 that is a factor of 25! is $(2 \cdot 5)^6 = 10^6$. So, $10^k = 10^6$ and k = **6**.

8. The smallest addend that is a multiple of 15 is 5! = 15 · 8. Since 5! is a factor of each addend greater than or equal to 5!, it follows that 5! + 6! + 7! + · · · + 49! + 50! = 15n, for some integer n, and, therefore,1! + 2! + 3! + 4 + 5! + 6! + 7! +· · · + 49! + 50! = 1! + 2! + 3! + 4! + 15n. The remainder when 1! + 2! + 3! + 4! + 15n is divided by 15, then, is equal to the remainder when 1! + 2! + 3! + 4! = 1 + 2 + 6 + 24 = 33 is divided by 15. We know that 33 = 2 · 15 + 3, so that remainder is **3**.