



An arithmetic sequence is a sequence of numbers in which the difference between each term and the next is always the same. Each of the following is an arithmetic sequence:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10$$

$$38, 34, 30, 26, 22, 18, 14, 10, 6, 2, -2$$

$$\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \frac{3}{2}, \frac{11}{6}, \frac{13}{6}, \frac{5}{2}$$

We can use “...” to say that a sequence continues indefinitely, so “1, 2, 3, 4, 5, 6, ...” refers to the arithmetic sequence of all positive integers. We say that the constant difference between two consecutive terms is the “common difference.” Thus, our three example sequences above have common differences 1,  $-4$  and  $\frac{1}{3}$ , respectively. Try the following problems before watching the lesson.

- Find the missing terms in each of the following arithmetic sequences:
  - 5, 11, \_\_, \_\_, \_\_
  - \_\_, \_\_, \_\_, 5, 11
  - 5, \_\_, \_\_, \_\_, 11
- An arithmetic sequence with 14 terms has 31 and 33 as its middle two terms. What is the largest term of the sequence?
- Suppose 8 is the first term and 3 is the common difference of an arithmetic sequence.
  - What is the 5<sup>th</sup> term of the sequence?
  - What is the 35<sup>th</sup> term of the sequence?
- Suppose the first term of an arithmetic sequence is  $a$  and the common difference is  $d$ . Find an expression for the  $n^{\text{th}}$  term in terms of  $a$ ,  $d$ , and  $n$ . Test your expression by making sure it gives the same answers you found for the previous problem.



## The Problems

**First Problem:** Consider an arithmetic sequence with  $a_3 = 165$  and  $a_{12} = 615$ . For what value of  $n$  is  $a_n = 2015$ ?

**Second Problem:** An arithmetic sequence has first term  $a$  and common difference  $d$ . If the sum of the first ten terms is half the sum of the next ten terms, what is the ratio  $\frac{a}{d}$ ? Express your answer as a common fraction.



## Follow-up Problems

5. If the second term of an arithmetic sequence is  $-7$  and the fifth term is  $106$ , then what is the  $14^{\text{th}}$  term?
6. Let  $v, w, x, y, z$  be an arithmetic sequence. If  $\frac{y}{w} = 3$ , then what is  $\frac{z}{x}$ ?
7. The sum of the first five terms of an arithmetic sequence is  $75$ . The sum of the first eleven terms of this sequence is  $363$ . What is the first term of the sequence?
8. The degree measures of the interior angles of a convex octagon form an increasing arithmetic sequence of positive integers. How many such sequences are possible?



## Share Your Thoughts

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