Activity Sheet for the November, 2015, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

1. Without writing anything, find the following:
   (a) The largest integer whose square is less than one million.
   (b) The least positive three-digit integer that leaves a remainder of 1 when divided by 7.
   (c) The least positive four-digit integer that leaves a remainder of 5 when divided by 9.

2. The product of three consecutive prime numbers is 2431. What is their sum?

3. What is the units digit of the product $1 \times 3 \times 5 \times \cdots \times 2015$?

4. Consider any positive three-digit integer that has all of its digits distinct and none equal to zero. What is the largest possible difference between such an integer and any integer that results from rearranging its digits?

First Problem: What is the units digit of the sum of the squares of the integers from 1 to 2015, inclusive?

Second Problem: The units and tens digits of one two-digit integer are the tens and units digits of another two-digit integer, respectively. If the product of the two integers is 4930, what is their sum?

Third Problem: If $p$ is the greatest prime whose digits are distinct prime numbers, what is the units digit of $p^2$?
5. What is the units digit of the product $2^{2015} \times 7^{2015}$?

6. What positive four-digit integer has its thousands and hundreds digits add up to the tens digit, its hundreds and tens digits add up to its ones digit and its tens and ones digits add up to the two-digit number formed by the thousands and hundreds digits?

7. What is the units digit of the sum of the cubes of the integers from 1 to 2015, inclusive?

8. If $2015 + a = b$ for positive integers $a$ and $b$, both of which are palindromes, what is the smallest possible value of $a$?

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).