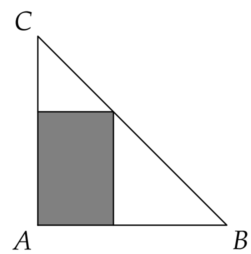
Warm-Up!

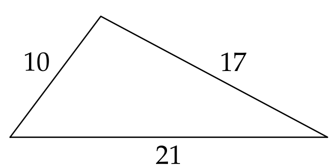
Try this problem before watching the lesson.

1. Triangle ABC is an isosceles right triangle with vertex at A such that each leg has length 6. What is the largest possible area of a rectangle that has one vertex at A, and each of the other three vertices is on a side of the triangle? One such rectangle is shown below.



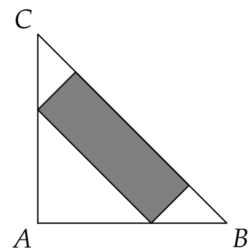
The Problem

In square units, what is the largest possible area a rectangle inscribed in the triangle shown here can have?



Follow-up Problems

2. Triangle ABC is an isosceles right triangle with vertex at A such that each leg has length 6. What is the largest possible area of a rectangle that has two vertices on the hypotenuse, and each of the other two vertices is on a leg of the triangle? One such rectangle is shown below.



Compare your answer to your solution to problem 1. Notice anything interesting? Is it a coincidence?

3. In rectangle $ABCD$, we have $AB = 6$ units and $BC = 12$ units. In square units, what is the the largest possible area a triangle inscribed in the rectangle can have?

4. Use the method shown in the video to find the area of a triangle whose sides have lengths 13, 14, and 15.

5. Use the method shown in the video to find the area of a triangle whose sides have lengths 5, 6, and 7.

Further Exploration

6. Heron's formula states that if s is half the perimeter of a triangle and the side lengths of the triangle are a, b, and c then the area of the triangle is

$$\sqrt{s(s-a)(s-b)(s-c)}.$$

Use the video's method for finding the height of a triangle to prove Heron's formula.

Wow! Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).