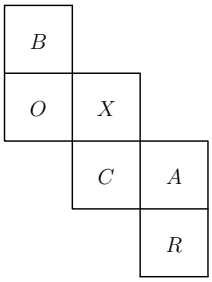
Activity Sheet for the May, 2017, MATHCOUNTS Mini

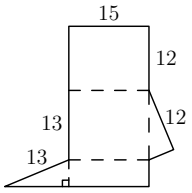
Warm-Up!

Try these problems before watching the lesson.

1. When this net is folded into a cube, what letter is on the face opposite the face labeled X?



2. The pattern, shown, is folded along the dashed lines to make a right triangular prism. What is the volume, in cubic units, of the triangular prism?



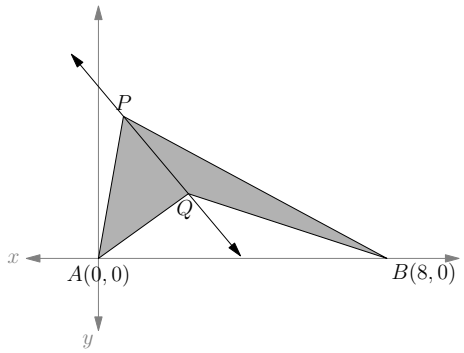
3. Suppose $\overline{AB}$, $\overline{AC}$, and $\overline{AD}$ are edges of a cube that has side length 6 cm. What is the volume of tetrahedron $ABCD$?

4. P.J. has a cylindrical mug with a 3-inch diameter. His mug is filled to a height of 6 inches with grape juice. Schuyler has a cylindrical mug with a 4-inch diameter. To what height in inches must Schuyler's mug be filled so that he receives the same amount of juice as P.J.?

The Problems

First Problem: A 12-sided game die has the shape of a hexagonal bipyramid, which consists of two pyramids, each with a regular hexagonal base of side length 1 cm and with height 1 cm, glued together along their hexagons. When this game die is rolled and lands on one of its triangular faces, how high off the ground is the opposite face?

Second Problem: Quadrilateral $APBQ$, shown here, has vertices $A(0,0)$ and $B(8,0)$, and vertices P and Q lie on the line given by the equation $4x+3y=19$. If $PQ=3$ units, what is the area of quadrilateral $APBQ$?



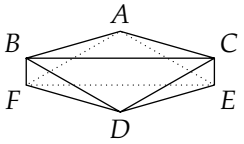
Follow-up Problems

5. Four 1-cm cubes are joined face-to-face in all possible ways to form geometric solids. Two such solids are considered the same if one can be obtained from the other by rotation. How many such solids are possible?

6. A gecko is in a room that is 12 feet long, 10 feet wide, and 8 feet tall. The gecko is currently on a side wall (10′ by 8′), one foot from the ceiling and one foot from the back wall (12′ by 8′). The gecko spots a fly on the opposite side wall, one foot from the floor and one foot from the front wall. What is the length of the shortest path the gecko can take to reach the fly assuming that it does not jump and can only walk across the ceiling and the walls?

7. A right cylindrical oil tank is 15 feet tall and its circular bases have diameters of 4 feet each. When the tank is lying flat on its side (not on one of the circular ends), the oil inside is 3 feet deep. How deep, in feet, would the oil have been if the tank had been standing upright on one of its bases? Express your answer as a decimal to the nearest tenth.

8. Faces ABC and DEF of the polyhedron below are parallel equilateral triangles with side length $4\sqrt{2}$ units. Each of the other edges in the polyhedron has length 4 units (i.e. $AE = EC = CD = DB = BF = FA = 4$). Find the volume of the polyhedron.



Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).