Activity Sheet for the May, 2015, MATHCOUNTS Mini

Warm-up!

Try these problems before watching the lesson.

1. In \(\triangle ABC\), side \(AB\) has length 4, and \(\angle A = \angle C = 45^\circ\). Find \(BC\) and \(AC\).

2. Point \(A\) is on segment \(BC\) such that \(BA : AC = 3 : 2\). If \(BC = 45\), then what is the length of \(AC\)?

3. In the diagram below, triangles \(RTS\) and \(QTP\) are similar. If \(PQ = 9\), \(RS = 6\), and \(SP = 10\), then what are \(PT\) and \(ST\)?

First Problem: Triangles \(ABD\) and \(DEF\) are isosceles right triangles. Points \(A, D, F\) and \(C\) are collinear, and points \(B, E\) and \(C\) are collinear. If \(AB = BD = 4\) and \(ED = EF = 2\), what is the length of segment \(AC\)?
Second Problem: In square $ABCD$, each vertex is connected to the midpoints of its two opposite sides, as shown. What is $QR/PQ$?

Follow-up Problems

4. In rectangle $ABCD$, $AB = 6$ units, the measure of $\angle DBC$ is $30^\circ$, $M$ is the midpoint of segment $AD$ and segments $CM$ and $BD$ intersect at point $K$. What is the length of segment $MK$?
5. In the diagram below, we have $DE = 2EC$ and $AB = DC = 20$. Find the length of $FG$.

6. Three coplanar squares with sides of lengths two, four, and six units, respectively, are arranged side-by-side, as shown so that one side of each square lies on line $AB$ and a segment connects the bottom left corner of the smallest square to the upper right corner of the largest square. What is the area of the shaded quadrilateral?

7. A square and isosceles triangle of equal height are side-by-side, as shown, with both bases on the $x$-axis. The lower right vertex of the square and the lower left vertex of the triangle are at $(10,0)$. The side of the square and the base of the triangle on the $x$-axis each equal 10 units. A segment is drawn from the top left vertex of the square to the farthest vertex of the triangle, as shown. What is the area of the shaded region?

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).