Activity Sheet for the December, 2015, MATHCOUNTS Mini
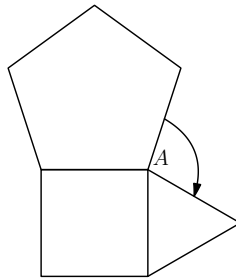
Warm-Up!

Try these problems before watching the lesson.

1. In $\triangle XYZ$, we have $\angle X = 30°$, $\angle Y = 90°$, and $YZ = 6$. Find XY and XZ.

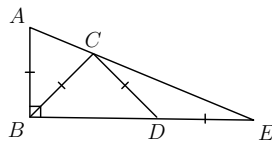2. Three regular polygons have common sides and vertices, as shown. What is the degree measure of angle A?

3. The area of $\triangle STU$ is 45. Points P and Q are on sides $\overline{ST}$ and $\overline{SU}$, respectively, such that $\overline{TU} \parallel \overline{PQ}$. If $SP = 2PT$, what is the area of $\triangle SPQ$?

4. Triangle ABC is an isosceles right triangle with vertex at A such that each leg has length 6. What is the largest possible area of a rectangle that has one vertex at A, and each of the other three vertices is on a side of the triangle? One such rectangle is shown below.
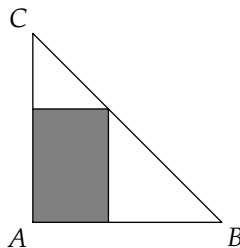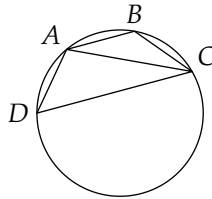
The Problems

First Problem: What is the radius of a circle inscribed in a triangle with sides of length 5, 12 and 13 units?

Second Problem: Points C and D are chosen on the sides of right triangle ABE, as shown, such that the four segments $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DE}$ each have length 1 inch. What is the measure of angle BAE, in degrees?
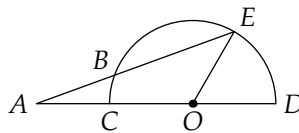
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Follow-up Problems

5. Two circles with radii 16 and 9 are tangent to each other, and are tangent to line ℓ at distinct points P and Q. Find the length of $\overline{PQ}$.

6. Triangle PQR is a right triangle with $\angle Q = 90°$, $PQ = 3$ and $QR = 4$. Points S, T and U are on sides $\overline{PQ}$, $\overline{PR}$ and $\overline{QR}$, respectively, such that $QSTU$ is a square. Find the length of $\overline{ST}$. Express your answer as a common fraction.

7. In the figure, $\overline{AB} \parallel \overline{CD}$, $m\angle ADC = 50$ degrees and $m\angle BAC = m\angle BCA$. What is the measure of $\angle BAD$?

8. In the figure, arc CBD is a semicircle with center O and diameter $\overline{CD}$. If $AB = OD$ and the measure of angle EOD is 60 degrees, what is the measure of angle A?

Wow! Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).