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Activity Sheet for the December, 2014, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

1. To express 20 as a sum of different powers of 2, we would write $20 = 2^4 + 2^2$. The sum of the exponents of these powers is $4 + 2 = 6$. If 400 were expressed as a sum of at least two distinct powers of 2, what would be the least possible sum of the exponents of these powers?
2. What is the base 4 representation of the base 2 number 11011000_2?
3. How many integers n from 1 to 100 are there such that $(n^2 - 15)/2$ is an even integer?

The Problems

First Problem: In base 5, what is the value of $27_{10} \times 314_5$?

Second Problem: How many digits are in the integer representation of 2^{30}?

Third Problem: Let $f(n) = \begin{cases} n^2 + 1 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$.

For how many integers n from 1 to 100, inclusive, does $f(f(\ldots f(n))) = 1$ for some number of applications of f?

Follow-up Problems

4. The base 9 representation of a positive integer is AB and its base 7 representation is BA. What is the integer expressed in base 10?

5. What is the tens digit in the decimal form of 11^8?

6. Zan has created this iterative rule for generating sequences of whole numbers:

   1) If a number is 25 or less, double the number.

   2) If a number is greater than 25, subtract 12 from it.

   Let F be the first number in a sequence generated by the rule above. F is a "sweet number" if 16 is not a term in the sequence that starts with F. How many of the whole numbers 1 through 50 are "sweet numbers"?

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).