Warm-Up!

Try these problems before watching the lesson.

1. Without writing anything, find the following:
   (a) The largest integer whose square is less than one million.
   (b) The least positive three-digit integer that leaves a remainder of 1 when divided by 7.
   (c) The least positive four-digit integer that leaves a remainder of 5 when divided by 9.

2. If the pattern shown continues, what is the letter in the 2010th position?
   MATHLETEMATHLETEMATHLETE...

3. Find the largest three-digit integer that leaves a remainder of 7 when divided by 9 and a remainder of 3 when divided by 11.

4. What is the remainder when $3^{2008}$ is divided by 5?

The Problems

First Problem: If 738 consecutive integers are added together, where the 178th number in the sequence is 4,256,815, what is the remainder when this sum is divided by 6?

Second Problem: For each positive integer $n$, $a_n = 9n + 2$ and $b_n = 7n + 3$. If the values common to both sequences are written as a sequence, the $n$th term of that sequence can be expressed as $pn + q$. What is the value of $p - q$?
Follow-up Problems

5. Find the least positive integer that leaves a remainder of 3 when divided by 5, a remainder of 7 when divided by 9, and a remainder of 7 when divided 13.

6. Find the least positive integer that leaves a remainder of 1 when divided by 2, a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, a remainder of 4 when divided by 5, a remainder of 5 when divided by 6, and a remainder of 6 when divided by 7.

7. A stadium staircase with 150 steps is numbered from 1 to 150. Jan starts on step 130 and steps down to step number 127, then to step number 124, and continues downward to step 13 stepping only on every third step. Jen starts on step number 3 and steps up to step number 7, then to step number 11, and continues upward to step 139 stepping only on every fourth step. How many steps were stepped on by both Jan and Jen (not necessarily at the same time)?

8. $x$, $y$ and $z$ are positive odd integers. What is the remainder when $x^2 + y^2 + z^2$ is divided by 4?

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).