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MATHCOUNTS®

2018
■ State Competition ■
Team Round
Problems 1–10

School ______________________________

Chapter ______________________________

Team Members ______________________________, Captain

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DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

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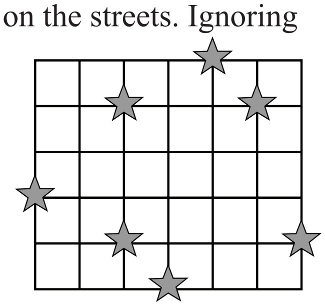
1. ______________ What is the arithmetic mean of the squares of the first ten positive integers? Express your answer as a decimal to the nearest tenth.

2. ______________ Two letters are randomly chosen without replacement from the word MATHCOUNTS. What is the probability that both are consonants? Express your answer as a common fraction.

3. ______________ Real numbers m and n exist such that $(n + 2)^2 - (n - 2)^2 = (m + 1)^2 - (m - 1)^2$. If m and n are nonzero and $m = an$, what is the value of a?

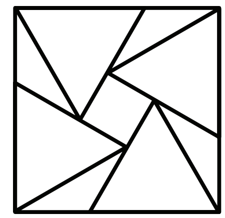
4. ____________ km The grid shows the streets in downtown Geometrika. All streets run either north-south or east-west. Adjacent parallel streets are 1 km apart. The seven restaurants in Geometrika are located at the street corners labeled with stars. To get between any two locations the city, Henry must walk on the streets. Ignoring street widths, he positions himself on the streets such that he minimizes the average of the walking distances between himself and the restaurants in the city. From Henry's chosen position, what is the average of the walking distances between him and each restaurant? Express your answer as a common fraction.



5. ____________ percent Jackie sold two cars for \$25,000 each. The first car sold for a profit of 22%, and the second sold at a loss of 7%. What was the total percent profit on the sale of the two cars? Express your answer to the nearest hundredth.

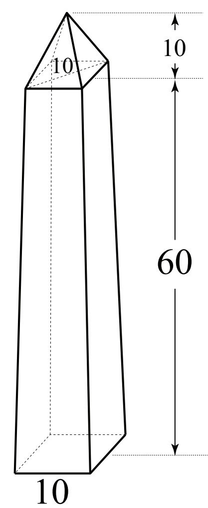
6. ______________ If A and B represent digits such that $\frac{A}{B} = \frac{A6}{6B} = \frac{A66}{66B} = \frac{A666}{666B} = \frac{A6666}{6666B}$, what is the sum of all possible digits A?

7. ______________ units A unit square contains four congruent non-overlapping equilateral triangles as shown in the figure. What is the largest possible side-length of one of the triangles? Express your answer as a decimal to the nearest thousandth.



8. ______________ Fabian has a deck of 25 cards. Each card has an integer between 1 and 5, inclusive, printed on it in one of five colors: red, orange, green, blue, or violet. Each number-color combination appears on exactly one card in the deck. Fabian draws four cards at random, without replacement, from the deck. What is the probability that exactly two different numbers and exactly three different colors appear on his four cards? Express your answer as a common fraction.

9. ______________ m^3 An *obelisk* is the frustum of a square pyramid, capped by a smaller square pyramid called the *pyramidion*. The obelisk shown has the following dimensions: the side length of the lower base of the frustum is 10 meters, the diagonal of the upper base of the frustum is 10 meters long, and the height of the frustum is 60 meters. The base of the pyramidion is congruent with the upper base of the frustum, and the height of the pyramidion is 10 meters. What is the total volume of the obelisk? Express your answer to the nearest ten.



10. ______________ If a certain sequence $a_1, a_2, a_3, a_4, \ldots$ of positive integers has the following properties, what is the greatest possible value of a_{99}?

For every positive integer k, $a_k < a_{k+1}$.
For every positive integer $k > 3$, $a_{k-3} + a_{k-2} + a_{k-1} + a_k = k^2$.