MATHCOUNTS®

2017
■ **State Competition** ■
Team Round
Problems 1–10

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School _____

Chapter _____

Team
Members _____ , Captain

_____

_____

_____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

Raytheon

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03-S17TEA

1. _____ The product of three consecutive integers is 157,410. What is their sum?

2. _____ kg Lorie wants to create a coffee blend that has regular coffee mixed with premium coffee in a ratio of 4:1. She starts with 100 kg of blend in which the ratio of regular to premium coffee is 3:2. How many kilograms of regular coffee does she need to add to get her desired 4:1 ratio?

3. _____ triples How many ordered triples of integers (x, y, z) satisfy $x^2 + y^2 + z^2 < 3$?

4. _____ Triangle PQR has vertices P(12, 10), Q(1, 3) and R(4, 9). An altitude of triangle PQR lies on a line that passes through point P and is perpendicular to QR. What is the y-intercept of that line?

5. _____ If a, b, c and d are nonnegative integers such that $\dfrac{a!b!}{c!d!} = 420$, what is the least possible value of $a + b + c + d$?

6. _____ ways   In Dualia, citizens pay for goods using only one-dollar bills, two-dollar bills and four-dollar bills. How many ways are there for a citizen to pay for an antique abacus that costs 2017 dollars using one or more of these three denominations?

7. _____ degrees
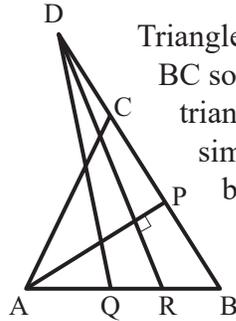
Triangle ABC, shown here, is isosceles with AC = BC. P lies on side BC so that the segment AP bisects angle BAC. D is located outside triangle ABC on ray BC. Q is on side AB so that triangle DQB is similar to triangle APB. R lies on side AB so that the segment DR bisects angle QDB. If the angle bisectors AP and DR intersect at a right angle, as shown, what is the degree measure of angle ABC? Express your answer as a common fraction.

8. _____   Each circle in the figure is to contain a distinct positive integer so that the numbers on each edge of the triangle form an arithmetic sequence in either increasing or decreasing order. Let x represent the greatest integer in the completed figure. What is the least possible value of x?
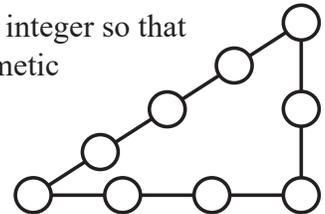
9. _____ feet   Eddie and Missy are swimming laps in parallel lanes of a swimming pool at different constant speeds. They start simultaneously at opposite ends of the pool. They first pass each other when Eddie has swum 72 feet. Both turn back when they reach the opposite ends, and they next pass each other when Eddie is 40 feet from Missy's starting point. What is the length of a lap?

10. _____ units²   In right triangle ABC, AB = 5, BC = 12 and AC = 13. P lies on side AC so that ray BP bisects angle ABC. Q and R lie on side BC so that BQ = QR = RC. Segments AQ and AR intersect segment BP at X and Y, respectively. What is the area of quadrilateral QRYX? Express your answer as a common fraction.