MATHCOUNTS®

2017
■ State Competition ■
Sprint Round
Problems 1–30

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HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

Raytheon

03-S17SPR

1. _____ inches   If the total area enclosed by the seven congruent squares shown is 112 in², what is the perimeter of the outer polygon?
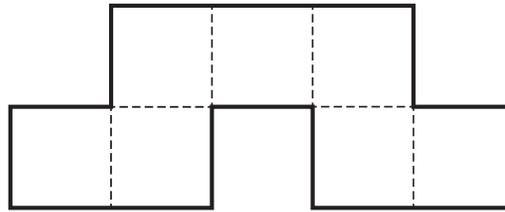

2. _____   Sasha added all but one of the first ten positive integers together. Her sum is a square number. Which integer did Sasha not include?

3. _____   If $\dfrac{x+1}{x} = 2$, what is the value of $\dfrac{x^2+1}{x^2}$?

4. $ _____   For every $50 worth of supplies Bill buys at a local hardware store, he receives a $5 discount. If Bill bought $846 worth of supplies from this store, what was the cost after the discounts were applied?

5. _____   Ayasha, Beshkno, and Chenoa were all born after 2000. Each of them was born in a year after 2000 that is divisible by exactly one of the prime numbers 2, 3 or 5. Each of these primes is a divisor of one of the birth years. What is the least possible sum of their birth years?

6. _____ degrees
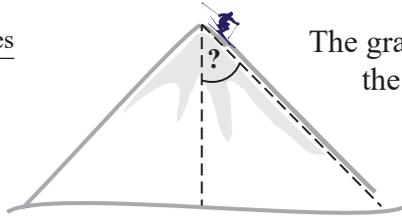
The grade of a ski slope is the ratio of the vertical drop to the horizontal distance. What is the measure of the angle between the ski slope and the vertical, as shown, of a ski slope whose grade is 100%?

7. _____ percent

Jeff has been collecting string and has wound it into a sphere with a diameter of 1.2 meters. He has collected enough additional string to increase the diameter of the sphere by 0.2 meters. What is the percent increase in the radius of the sphere? Express your answer as a decimal to the nearest tenth.

8. _____ baskets

Allie plays basketball with her friends and makes 10 baskets. If each basket is worth either 2 or 3 points, and Allie scores a total of 26 points, how many baskets worth 3 points did she make?

9. _____ cm^3

A small cube is built inside a large cube as shown. The edge length of the large cube is four times the edge length of the small cube. If the small cube has volume 5 cm^3, what is the volume of the space inside the large cube surrounding the small cube?
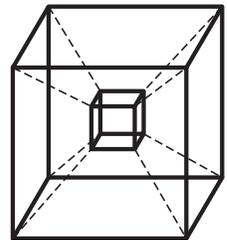

10. _____

Andre can complete $\frac{5}{6}$ of a job in $\frac{3}{4}$ of the time that it takes Michael to do the whole job. What is the ratio of the rate at which Andre works to the rate at which Michael works? Express your answer as a common fraction.

11. _____ grains   Eleven boastful bees are all lined up in a row. Each bee, after the first one, brags that it collected one more than twice as many grains of pollen as the bee in front of it. If the first bee has 100 grains of pollen, how many grains did the last bee collect?

12. _____   The mean of a list of nine numbers is 17, and the modes are a, b and c. If $a + 4$, $1 + b$ and $c - 8$ are distinct numbers in the list, and none of them are modes of the list, then what is the value of $3(a + b + c)$?

13. _____ in²   The perimeter of a rectangle is 40 inches and each diagonal of the rectangle is 18 inches long. What is the area of this rectangle?

14. _____ ways   The figure shows five rows, each containing five bricks. In how many ways can Mario choose five bricks, one in each row, so that any two bricks chosen in adjacent rows are adjacent?
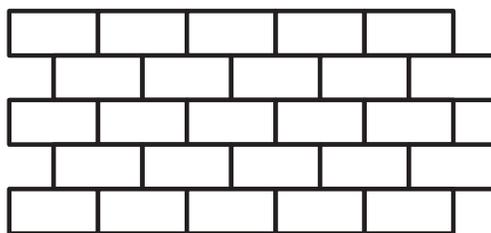
15. _____   What is the least positive base-10 integer that can be written as a 4-digit number in base 3 and as a 3-digit number in base 4?

16. _____ If x and y are integers such that $(x-3)^2 + (y+4)^2 = 25$, what is the greatest possible value of $x^2 + y^2$?

17. _____ A positive integer q is the product of a prime number and a perfect square. Additionally, q is the product of a different prime number and a perfect cube. What is the least possible value of q?

18. _____ If $\dfrac{3}{2-x} + \dfrac{2}{y+3} = 1$ and $\dfrac{2}{2-x} + \dfrac{8}{y+3} = 2$, what is the value of x?

19. _____ A pole of radius 1 inch and height h inches is perpendicular to the ground. A snake is coiled around the pole in a spiral pattern, making a 60-degree angle with the ground. If the snake is 9π inches long and reaches from the ground to the top of the pole, then h can be written in the form $a\pi\sqrt{b}$, where a is a rational number and b is a positive integer that is not divisible by the square of any prime. What is the value of $a + b$? Express your answer as a common fraction.

20. _____ The median of the five numbers $2^{(3^4)}$, $2^{(4^3)}$, $3^{(2^4)}$, $4^{(3^3)}$ and $4^{(2^3)}$ is equal to p^k for some prime number p and some positive integer k. What is the value of $p + k$?

21. _____ numbers   The number 40,231 is a five-digit positive integer that uses five consecutive digits, although not necessarily in order. How many such five-digit numbers are there?

22. _____ inches   Chloe finds a coin, as shown, wedged tightly into the corner of her drawer behind a rectangular box with a 2.5-inch edge. One corner of the box is 1.5 inches from the corner of the drawer and the other corner of the box is 2 inches from the corner of the drawer. What is the diameter of the coin?

23. _____   What four-digit number has tens digit 2 and units digit 8, is a multiple of 16, and when its digits are reversed the result is also a multiple of 16?

24. _____ units   What is the radius of the inscribed circle of a triangle with side lengths 9, 13 and 14? Express your answer in simplest radical form.

25. _____   On an infinitely large grid of squares, a two by two region is shaded. Starting in the square above the upper left shaded square, the squares in the grid are numbered with consecutive positive integers, moving in a clockwise direction to the next open square. The figure shows the grid with the first 18 squares numbered. What integer will appear in the 6th square to the right of the square numbered 5?

26. _____ A triangle in the coordinate plane has vertices at $(-4, 0)$, $(6, 0)$ and $(0, 5)$. The line $y = \dfrac{5}{4}x + c$, where c is a positive number, divides the triangle into a trapezoid and a smaller triangle whose areas, respectively, are in the ratio 5:4. What is the value of c? Express your answer as a common fraction.

27. _____ ways In how many ways can 20 identical coins be given to 4 distinct people so that each person gets at least 2 coins?

28. _____ What is the sum of all values of x that satisfy the equation $\dfrac{x^{x-3}}{x} = \dfrac{x}{x^{\frac{4}{x}}}$?

29. _____ Lines AB and DC are parallel, and transversals AC and BD intersect at a point X between the two lines so that $\dfrac{AX}{CX} = \dfrac{5}{7}$. Points P and Q lie on segments AB and DC, respectively. The segment PQ intersects transversals BD and AC at points M and N, respectively, so that $PM = MN = NQ$. What is the ratio $\dfrac{AP}{BP}$? Express your answer as a common fraction.

30. _____ Suppose that a is a number such that the system of equations
$$|2x| - y = 5$$
$$x - |2y + 2| = a$$
has an odd number of distinct solutions. What is the product of all possible values of a?

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and $GCF(a, b) = 1$. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and $GCF(a, b) = 1$. Examples:

Problem: What is $8 \div 12$ expressed as a common fraction?   *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is $12 \div 8$ expressed as a common fraction?   *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle with diameter $\frac{1}{4}$ unit

　　　expressed as a common fraction in terms of π?   *Answer:* $\frac{1+2\pi}{8}$

Problem: What is $20 \div 12$ expressed as a mixed number?   *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$　　　*Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is the value of $\sqrt{15}\times\sqrt{5}$?   *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* both are zero, in which case they both may be omitted. Answers in the form ($)*a.bc* should be rounded to the nearest cent unless otherwise specified. Examples:

Acceptable: 2.35, 0.38, .38, 5.00, 5　　　　　*Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not perform any intermediate rounding (other than the "rounding" a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples:

Problem: What is 6895 expressed in scientific notation?   *Answer:* 6.895×10^3

Problem: What is 40,000 expressed in scientific notation?   *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.
Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.