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# MATHCOUNTS®

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2017  
■ State Competition ■  
Countdown Round  
Problems 1–80

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This booklet contains problems to be used  
in the Countdown Round.

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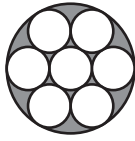
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1. \_\_\_\_\_ If 60% of the students in a class are boys, what is the ratio of girls to boys?  
Express your answer as a common fraction.

2. \_\_\_\_\_ In the figure, six of the seven congruent inner circles are tangent to the outer circle, to the central circle and to two other adjacent circles. What fraction of the largest circle's area is gray? Express your answer as a common fraction.



3. \_\_\_\_\_ What is the sum of all values of  $x$  such that the parabola  $y = \frac{2}{3}x^2 + 7x + 14$  passes through the point  $(x, \frac{32}{3})$ ? Express your answer as a common fraction.

4. \_\_\_\_\_ If the 6-digit number  $82N281$  is equal to  $909^2$ , what digit does  $N$  represent?

5. \_\_\_\_\_ (numbers) How many 4-digit numbers are there in base 5?

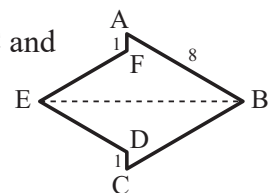
6. \_\_\_\_\_ ( $m^3$ ) The surface area of a cube is  $54 m^2$ . What is its volume, in cubic meters?

7. \_\_\_\_\_ (dollars) Alton's bank will transfer money from his account to Justin's account at another bank but, as a fee, will deduct 10% of the amount of money transferred. To the nearest dollar, how much must Alton transfer so that Justin receives \$3000?

8. \_\_\_\_\_ What is the value of  $\frac{2018!}{2016! + 2017!}$ ?

9. \_\_\_\_\_ (people) Of 1000 people surveyed, each person owned a cat, a dog or both. One-third of the 630 people who owned a cat also owned a dog. How many of the people surveyed owned a dog?

10. \_\_\_\_\_ (units) The polygon shown is composed of equilateral triangles  $ABC$  and  $DEF$ . If  $AB = 8$  units and  $FA = CD = 1$  unit, what is  $BE$ , in units? Express your answer in simplest radical form.



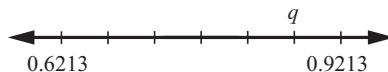
11. \_\_\_\_\_ What is the units digit of  $4^{2017} - 3^{2017}$ ?

12. \_\_\_\_\_ What integer is equivalent to  $(\frac{1}{2})^{-3}$ ?

13. \_\_\_\_\_ If  $(a^{\sqrt{a}})^2 = 256$ , what is the value of  $a$ ?

14. \_\_\_\_\_ What is the greatest negative integer solution to the absolute value inequality  $\left| \frac{2x-7}{5} \right| > 4$ ?

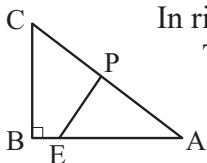
15. \_\_\_\_\_ A portion of a number line is divided into six equal parts as shown. What is the value of  $q$ ? Express your answer as a decimal to the nearest ten-thousandth.



16. \_\_\_\_\_ (combinations) A milkshake is made by blending three of five flavors: chocolate, vanilla, strawberry, cherry and peach. How many different combinations of three distinct flavors are possible?

17. \_\_\_\_\_ What value of  $n$  makes the equation  $n^{n-1} + (n-1)^n = 145$  true?

18. \_\_\_\_\_ (units) In right triangle ABC, shown here, BA = 8 units and BC = 6 units. The perpendicular bisector of side AC intersects side AB at E. What is the length of segment AE, in units? Express your answer as a common fraction.



19. \_\_\_\_\_ What is the least common multiple of 6, 10 and 14?

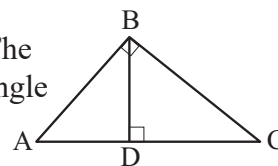
20. \_\_\_\_\_ (degrees) The interior angle measures in an octagon form an arithmetic sequence. The largest angle measures 156 degrees. What is the degree measure of the smallest interior angle?

21. \_\_\_\_\_ What is the greatest integer  $x$  for which  $(x^2)! \leq (x!)^2$ ?

22. \_\_\_\_\_ What is the sum of the numerical coefficients of the expansion of  $(2x + 2y)^5$ ?

23. \_\_\_\_\_ (years) The mean age of three students is 20 years. All three of the students are at least 18 years old. What is the greatest possible age of the oldest student, in years?

24. \_\_\_\_\_ (units) Right triangle ABC, shown here, has sides of integer length. The length of altitude BD is 12 units. What is the perimeter of triangle ABC, in units?



25. \_\_\_\_\_ If  $\frac{1}{6}$  of 400 equals  $\frac{5}{6}$  of  $x$ , what is the value of  $x$ ?

26. \_\_\_\_\_ (integers)  $C(19, k)$  denotes the number of combinations of  $k$  items chosen from a group of 19 items. For how many integers  $k$  from 0 to 19, inclusive, is  $C(19, k)$  divisible by 19?

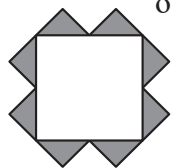
27. \_\_\_\_\_ The mean of six evenly spaced numbers is 2017. What is the sum of the mean and the median of the six numbers?

28. \_\_\_\_\_ A certain island has 14 towns. Each pair of towns is connected by a unique road, labeled with a positive integer. The road labels are consecutive positive integers beginning with 1. What is the greatest number used to label a road on the island?

29. \_\_\_\_\_ When  $\sqrt{0.005}$  is written as a decimal, what is the hundredths digit?

30. \_\_\_\_\_ If  $\frac{1}{16}$  of  $2^{40}$  equals  $4^k$ , what is the value of  $k$ ?

31. \_\_\_\_\_ The figure shows two gray rectangles and a white square overlapping. The sides of each rectangle intersect the four midpoints of the square's sides, and the vertices of the square intersect the midpoints of two sides of each rectangle, as shown. What is the ratio of the combined area of the gray regions to the area of the square? Express your answer as a common fraction.

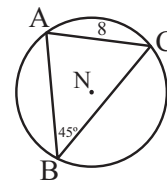


32. \_\_\_\_\_ What is the least value of  $z$  for which  $z!$  is a multiple of 50?

33. \_\_\_\_\_ Given that  $y$  varies inversely with  $x$ , and  $y = 13$  when  $x = 57$ , what is the value of  $y$  when  $x = 19.5$ ?

34. \_\_\_\_\_ Let  $a, b, c$  and  $d$  be distinct nonnegative integers such that  $a^2 - b^2 = c^2 - d^2$ . What is the least possible value of  $a + b$ ?

35. \_\_\_\_\_ (units<sup>2</sup>) Triangle  $ABC$  is inscribed in circle  $N$  as shown. If  $\angle ABC$  has measure 45 degrees and  $AC = 8$  units, what is the area of circle  $N$ , in square units? Express your answer in terms of  $\pi$ .



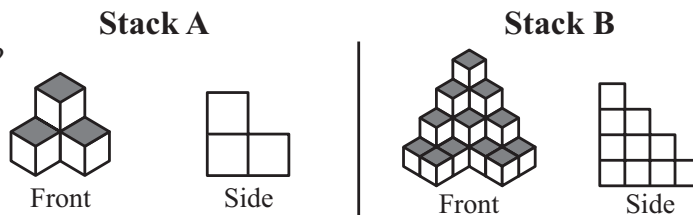
36. \_\_\_\_\_ What is the value of  $(37)(33) - (39)(31)$ ?

37. \_\_\_\_\_ If January 1st is on a Monday in a non-leap year, on what day of the week is December 11th in the same year?
38. \_\_\_\_\_ What is the greatest integer  $n$  for which  $2^n$  divides  $(10!)(9!)(8!)(7!)(6!)(5!)(4!)(3!)(2!)(1!)$ ?
39. \_\_\_\_\_ What is the value of  $\frac{1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3}{1 + 2 + 3 + 4 + 5 + 6}$ ?
40. \_\_\_\_\_ If  $f(n) = \frac{1}{n^2 + n}$ , what is the value of  $f(1) + f(2) + f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10)$ ? Express your answer as a common fraction.
41. \_\_\_\_\_ A circle of radius  $2\sqrt{2}$  units has center  $(0, 0)$ . A chord from  $(2, 2)$  to  $(2, -2)$  divides the circle into two regions. The area of the larger region can be expressed as  $a\pi + b$  units<sup>2</sup>. What is the value of  $ab$ ?
42. \_\_\_\_\_ Seven positive numbers form an arithmetic sequence, and their reciprocals form a geometric sequence. If the third term of the arithmetic sequence is 12, what is the greatest possible value of the seventh term?
43. \_\_\_\_\_ (units) What is the distance, in units, between the points  $(3, -5, 7)$  and  $(1, -2, 1)$  in three-dimensional space?
44. \_\_\_\_\_ The integers 1 to 9 are randomly placed into the nine squares of a  $3 \times 3$  array, one integer per square. What is the probability that the squares containing 1 and 2 share a side or a vertex with each other? Express your answer as a common fraction.
45. \_\_\_\_\_ If  $AB$  and  $BA$  are different two-digit numbers with the digits reversed and  $AB - BA = 36$ , what is the least possible value of the two-digit number  $AB$ ?
46. \_\_\_\_\_ (cm) Two circles, each with radius 6 cm, intersect at  $A$  and  $B$  such that the segment connecting their centers is divided into three congruent segments by the circles. What is the length, of the common chord  $AB$ , in centimeters? Express your answer in simplest radical form.
47. \_\_\_\_\_ Positive integers  $w, x, y$  and  $z$  exist such that  $\frac{x}{y} = \frac{2w}{z}$  and  $\frac{x}{z} = \frac{2y}{w}$ . What is the value of  $\frac{x}{y}$ ?

48. \_\_\_\_\_ What is the sum of all  $m$  for which the value of  $\frac{2}{m + \frac{3}{m+4}}$  is undefined?

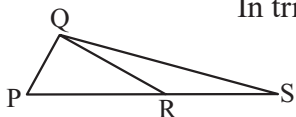
49. \_\_\_\_\_ What is the value of  $\left(1 + \frac{3}{4}\right)\left(1 + \frac{3}{5}\right)\left(1 + \frac{3}{6}\right)\left(1 + \frac{3}{7}\right)$ ?

50. \_\_\_\_\_ (in<sup>3</sup>) In the figure shown, the cubes in stack A have edge length 2 inches, and the cubes in stack B have edge length 1 inch. What is the absolute difference between the volumes of the



51. \_\_\_\_\_ (people) Three million people paid to watch a movie the weekend the DVD was released. Some paid \$6 to rent the movie, and the rest paid \$15 to buy it. The total payments made were 22.5 million dollars. How many people bought the movie?

52. \_\_\_\_\_ In triangle PQS, shown here,  $\angle PQR$  is a right angle,  $PR = 2PQ$  and  $QR = SR$ . What is the degree measure of  $\angle RQS$ ?



53. \_\_\_\_\_ What is the sum of the values of  $s$  for which  $\frac{5}{s} = \frac{s}{35}$ ?

54. \_\_\_\_\_ For positive number  $p$ , the product  $40p$  expressed as a decimal has a nonzero hundredths digit. What is the least possible value of  $p$ ? Express your answer as a common fraction.

55. \_\_\_\_\_ If  $n$  and  $m$  are numbers such that  $n(n + m) = 6$  and  $m(n + m) = 18$ , what is the value of  $nm$ ? Express your answer as a common fraction.

56. \_\_\_\_\_ (numbers) How many seven-digit numbers have seven distinct nonzero digits that appear in increasing order from left to right?

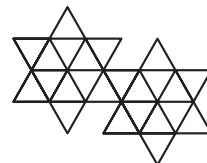
57. \_\_\_\_\_ What is the least possible value of  $a^{bc}$  where  $a$ ,  $b$  and  $c$  each represent a different number chosen from the set  $\{2, 3, 4\}$ ?

58. \_\_\_\_\_ What is the remainder when 38 dozen is divided by 7?

59. \_\_\_\_\_ (percent) A discount card offers \$10 off any purchase totaling \$100 to \$199.99 or \$25 off any purchase totaling \$200 or more. What is the maximum percent discount that can be obtained using this card? Express your answer to the nearest tenth.

60. \_\_\_\_\_ A line with slope  $\frac{1}{3}$  passing through  $(-3, 1)$  intersects a second line with slope  $-\frac{2}{3}$  at  $(6, a)$ . What is the  $y$ -coordinate of the  $y$ -intercept of the second line?

61. \_\_\_\_\_ (triangles) How many triangles of any size are in the figure shown here?



62. \_\_\_\_\_ The number 65,432 is a *countdown* number because each successive digit is one less than its predecessor. What is the sum of all 5-digit *countdown* numbers?

63. \_\_\_\_\_ Let  $(x^x)^{5x} = x^{x^5}$  for  $x > 1$ . If the value of  $x$ , in simplest radical form, is  $\sqrt[a]{b}$ , what is the value of  $a + b$ ?

64. \_\_\_\_\_ If  $5x + 2 = 4.003$ , what is the value of  $20x + 7$ ? Express your answer as a decimal to the nearest thousandth.

65. \_\_\_\_\_ (mL) The dosage for an antibiotic is 30 mg per kilogram of body weight. Every gram of antibiotic is dissolved in 10 mL of sterile water. In one dose, how many milliliters of the antibiotic solution should be administered to a child weighing 24 kg? Express your answer as a decimal to the nearest tenth.

66. \_\_\_\_\_ (cm) The length of a 12 cm segment is increased by 50%. The length of the resulting segment is then increased by 50%. After two additional increases, each by 50% of the resulting length, what is the length of the final segment, in centimeters? Express your answer as a common fraction.

67. \_\_\_\_\_ A sphere is inscribed in a cube. Let  $P$  be the ratio of the volume of the sphere to that of the cube, and let  $Q$  be the ratio of the surface area of the sphere to that of the cube. What is the value of  $\frac{P}{Q}$ ?

68. \_\_\_\_\_ (ordered pairs) How many ordered pairs of positive integers  $(a, b)$  are there such that  $ab = 1200$  and  $a - b$  is an odd positive integer?

69. \_\_\_\_\_ Two single-digit positive integers have a product of 72. What is their sum?

70. \_\_\_\_\_ What is the absolute difference between the mean and median of the three 9-digit numbers 981,457,623 and 981,457,625 and 981,457,622? Express your answer as a common fraction.

71. (April) \_\_\_\_\_ Maggie, the goat, gave birth to three kids on March 21. Abigail, the goat, gave birth to twins on April 5. On what date in April will the mean age of all five kid goats be 30 days old?


72. \_\_\_\_\_ What digit does A represent in the equation  $1A \times 491 = AA3A$ ?

73. \_\_\_\_\_ If  $f(x) = x^2 - 2x - 1$ , what is the minimum value of  $f(|x|)$ ?

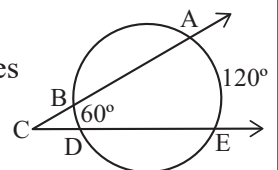
74. \_\_\_\_\_ In the sequence 1, 1, 2, 3, 5, ... each term after the second term is the sum of the previous two terms. The new sequence 1, 1, -2, 3, 5, ... is created by negating every third term of the original sequence. What is the sum of the first ten terms of this new sequence?

75. \_\_\_\_\_ After four tests, Lexa has a mean score of 88. What score does she need on the fifth test to raise her mean score to 90?

76. \_\_\_\_\_ Isosceles triangle ABC has a base of length  $5x - 8$  and legs of length  $6x - 5$ . What is the least possible integer value for  $x$ ?

77. \_\_\_\_\_ (units<sup>2</sup>)  Kite ABCD is inscribed in circle O. A radius and segments of the two diagonals of the kite form triangle OAE, as shown, with  $OE = 6$  units,  $AE = 8$  units and  $OA = 10$  units. What is the area of the kite, in square units?

78. \_\_\_\_\_ (degrees) In the figure, arc AE measures 120 degrees and arc BD measures 60 degrees. What is the degree measure of  $\angle C$ ?



79. \_\_\_\_\_ What is the sum of the infinite sequence  $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \dots$ , where each term after the first is half of the previous term?

80. \_\_\_\_\_ (years old) If three less than the product of 13 and Kris' current age equals the sum of three times his age 3 years from now and six times his age 6 years from now, how many years old will Kris be six years from now?