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# MATHCOUNTS®

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2017

■ **School Competition** ■  
**Team Round**  
**Problems 1–10**

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Team  
Members \_\_\_\_\_, Captain  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

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Total Correct	Scorer's Initials

## Raytheon

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1. \_\_\_\_\_ If  $11 \times n = 123,456,787,654,321$ , what is the value of  $n$ ?

2. ( \_\_\_\_\_ , \_\_\_\_\_ ) The point  $(4, -2)$  is reflected over the line  $y = x$ . What are the coordinates of its image? Express your answer as an ordered pair.

3. \_\_\_\_\_  The ages, in years, of four members of a family are represented by  $a, b, c$  and  $d$ , where  $a < b < c < d$ . Their mean age is 34, their median age is 33, and the range of their ages is 32. What is the value of  $a$ ?

4. \_\_\_\_\_ base 7 What is the value of  $32_4 + 43_5 + 54_6$  when written in base 7?

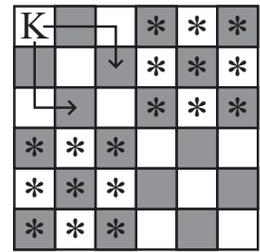
5. \_\_\_\_\_ Raquel uses six different digits to fill in the blanks below, writing one digit in each blank, so that the resulting addition statement is correct. What is the least possible sum of the six digits?

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

6. \_\_\_\_\_ If  $a, b$  and  $c$  satisfy the equations  $a^2 + b^2 = 313$ ,  $b^2 + c^2 = 277$  and  $a^2 + c^2 = 302$ , what is the value of  $a^2 + b^2 - c^2$ ?

7. \_\_\_\_\_ moves

A chess knight makes L-shaped moves on a grid of squares. During each move, the knight moves two squares either up, down, left, or right, then one square in a perpendicular direction. The knight starts on the square marked K in the 6-by-6 board shown here, and its two possible first moves are shown. What is the least number of moves the knight must make from the K in order to land at least once on each of the squares marked with a star?

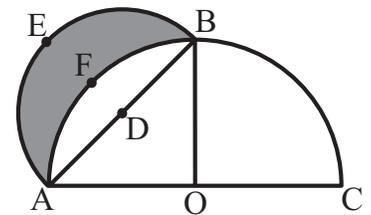


8. \_\_\_\_\_

A chess club has 8 girls and 6 boys. Two members, Zig and Zag, are fraternal twins of different genders. If a team of 3 girls and 3 boys is randomly selected for the district championship, what is the probability that exactly one of the twins is on the team? Express your answer as a common fraction.

9. \_\_\_\_\_ units<sup>2</sup>

Triangle ABO is an isosceles right triangle, AC is a diameter of circle O, and AB is a diameter of circle D. If the semicircle centered at O has area  $2\pi$ , what is the shaded area AEBF (which is called a *lune*)?



10. \_\_\_\_\_

Each cell in the 3-by-3 array of squares shown is filled with a digit from 1 to 9, inclusive. If the four listed conditions are satisfied, what is the resulting three-digit number when diagonal B is read from upper-left to lower-right?

- (1) Each digit from 1 through 9 is used exactly once.
- (2) Each row, read from left to right, forms an odd three-digit multiple of 3 that is not a multiple of 9.
- (3) Each column, read from top to bottom, forms an odd three-digit multiple of 3 that is not a multiple of 9.
- (4) Diagonal A, read from lower-left to upper-right, forms an odd three-digit multiple of 9.

