Warm-Up!

Try these problems before watching the lesson.

1. Each minute, Tina climbs up three stairs, and then down two stairs. After forty minutes, how many stairs higher is she than when she started?

2. Evaluate the sum

$$(4-3)+(5-4)+(6-5)+(7-6)+\cdots+(2010-2009).$$

3. Evaluate the product

$$\frac{5}{3}\cdot\frac{6}{4}\cdot\frac{7}{5}\cdot\frac{8}{6}\cdot\cdots\cdot\frac{2010}{2008}.$$

4. Evaluate each of the following three sums:

   (a) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}$

   (b) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}$

   (c) $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}$

   Use the results to guess the value of the sum

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots+\frac{1}{2009\cdot 2010}.$$

The Problem

What is the sum of the 2009 fractions of the form $\frac{2}{n(n+2)}$ if the values of n are the positive integers from 1 through 2009? Express your answer as a decimal to the nearest thousandth.

Follow-up Problems

5. Show that your guess to Problem 4 is correct.

6. What is the value of the infinite sum

$$\frac{2}{1(1+2)}+\frac{2}{2(2+2)}+\frac{2}{3(3+2)}+\frac{2}{4(4+2)}+\frac{2}{5(5+2)}+\cdots?$$

   (This is simply the infinite version of the original MATHCOUNTS problem.)

7. Find the smallest integer n for which the sum of the integers from -25 to n (including -25 and n) is at least 26.

8. (Extra challenging!) Evaluate the sum

$$\frac{3}{1^2\cdot 2^2}+\frac{5}{2^2\cdot 3^2}+\frac{7}{3^2\cdot 4^2}+\cdots$$

   Source: Harvard-MIT Mathematics Tournament

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).