Warm-Up!

Try these problems before watching the lesson.

1. In $\triangle ABC$, side $\overline{AB}$ has length 4, and $\angle A = \angle C = 45^\circ$. Find BC and AC.

2. In $\triangle XYZ$, we have $\angle X = 30^\circ$, $\angle Y = 90^\circ$, and $YZ = 6$. Find XY and XZ.

The Problem

Two angles of a triangle measure 30 and 45 degrees. If the side of the triangle opposite the 30-degree angle measures $6\sqrt{2}$ units, what is the sum of the lengths of the two remaining sides?

Follow-up Problems

3. Two angles of a triangle measure 45 and 105 degrees. If the side of the triangle opposite the 45-degree angle measures 8 units, what is the sum of the lengths of the two remaining sides?

4. Find a formula for the area of an equilateral triangle with side length x. Your formula should be in terms of x.

5. Two angles of a triangle measure 45 and 105 degrees. If the side of the triangle opposite the 105-degree angle measures 12 units, then what is the area of the triangle?

6. Two angles of a triangle measure 45 and 120 degrees. If the side of the triangle opposite the 45-degree angle measures $6\sqrt{3}$ units, then what is the length of the side opposite the 120-degree angle?

Further Exploration

Suppose a triangle has sides with lengths a, b, and c. Let $s = (a+b+c)/2$; we call this the "semiperimeter" of the triangle. A famous formula, called **Heron's formula**, tells us that the area of the triangle equals $\sqrt{s(s-a)(s-b)(s-c)}$.

7. Use Heron's formula to find the area of an equilateral triangle with side length 6. Compare your result with Heron's formula to the result given by the formula you found in Problem 4.

8. Use Heron's formula to find the area of a right triangle with legs 7 and 24.

9. Prove Heron's formula. (This is very challenging! The reason we include this problem with this video is that the first step in our solution in the video is also helpful here—right triangles are our friends!)

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).