Activity Sheet for the February, 2013, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

1. Two numbers are called **relatively prime** if their greatest common divisor is 1. For each of the following, state whether or not the two numbers are relatively prime:
   (a) 40 and 62
   (b) 123 and 321
   (c) $3 \cdot 5 \cdot 7 \cdot 9$ and $11 \cdot 13 \cdot 15 \cdot 17$
   (d) 1,234,567,890 and 1,234,567,891
2. How many two-element subsets are there of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$?
3. How many positive integers less than 20 are there such that 2 times the integer is a perfect square?

The Problem

First Problem: Products are found by multiplying three different numbers from the set $\{1, 2, 3, 4, 5\}$. Among those products, how many pairs of relatively prime numbers are there?

Second Problem: For how many two-element subsets $\{a, b\}$ of the set $\{1, 2, 3, \ldots, 36\}$ is the product ab a perfect square?

Follow-up Problems

4. How many positive integers less than 1000 are there such that 2 times the integer is a perfect square?

5. Products are found by multiplying three different numbers from the set $\{1, 2, 3, 4, 5, 6\}$. Among those products, how many different pairs of relatively prime numbers are there?

6. Products are found by multiplying three different numbers from the set $\{1, 2, 3, 4, 5, 6, 7\}$. Among those products, how many different pairs of relatively prime numbers are there?

7. How many 4-digit positive integers are there such that no two consecutive digits are equal?

8. For how many pairs of distinct positive integers a and b, both less than 100, is a/b the square of an integer?

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).