Warm-Up!
1. We are told that \( a = b^2 - 3 \). Since \( b^2 \) will always be non-negative, we will get the minimum value of \( a \) when \( b = 0 \). Therefore, the smallest possible value of \( a \) is \( a = 0^2 - 3 = -3 \).

2. (a) \((x + 1)^2 = (x + 1)(x + 1) = x^2 + x + x + 1 = x^2 + 2x + 1\)
   (b) \((x + 2)^2 = (x + 2)(x + 2) = x^2 + 2x + 2x + 4 = x^2 + 4x + 4\)
   (c) \((x + 3)^2 = (x + 3)(x + 3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9\)
In general, we see that \((x + y)^2 = x^2 + 2xy + y^2\) for any real numbers \( x \) and \( y \).

3. Using the above generalization, we have \((x + y)^2 = x^2 + 2xy + y^2 = x^2 + 14x + \_\_\_\_.\) It follows that \(2xy = 14x\), and \(y = 14x/2x = 7\). The value that completes the square is \(y^2 = 7^2 = 49\).

4. We are asked to expand \((a + b + c)^2\). We have \((a + b + c)^2 = (a + b + c)(a + b + c)\).
   Using the distributive property, we get \(a(a + b + c) + b(a + b + c) + c(a + b + c) = a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2 = a^2 + b^2 + c^2 + 2(ab + ac + bc)\).

The Problem is solved in the MATHCOUNTS Mini.

Follow-up Problems
5. We are told that \( x + y = 3 \) and \( x^2 + y^2 = 6 \), and then asked to find the value of \( xy \). Squaring both sides of the first equation, we get \((x + y)^2 = 3^2 \rightarrow x^2 + 2xy + y^2 = 9 \rightarrow x^2 + y^2 + 2xy = 9\). We can substitute 6 for \(x^2 + y^2\) to get \(6 + 2xy = 9 \rightarrow 2xy = 3 \rightarrow xy = 3/2\).

6. Let \(x = 20122011\), and let \(y = 20122009\). We are asked to evaluate the expression \(20122011^2 - 2(20122011)(20122009) + 20122009^2\), which can be rewritten as \(x^2 - 2xy + y^2\).
   Factoring, we see this is equivalent to \((x - y)^2\). So we have \((x - y)^2 = (20122011 - 20122009)^2 = 2^2 = 4\).

7. We are told that \(a + (1/a) = 6\) and asked to determine the value of \(a^4 + (1/a^4)\). Squaring the first equation yields \((a + (1/a))^2 = 6^2 \rightarrow a^2 + 2 + (1/a^2) = 36 \rightarrow a^2 + (1/a^2) = 34\). Now squaring each side of this equation, we get \((a^2 + (1/a^2))^2 = 34^2 \rightarrow a^4 + 2 + (1/a^4) = 1156\). Therefore, \(a^4 + (1/a^4) = 1154\).

8. We are told that \(xyz = 45\) and \(1/x + 1/y + 1/z = 1/5\). We can rewrite the left side of the second equation using a common denominator to get \((yz/xyz) + (xz/xyz) + (xy/xyz) = 1/5 \rightarrow (xy + xz + yz)/xyz = 1/5\). But we know that \(xyz = 45\), so we have \((xy + xz + yz)/45 = 1/5 \rightarrow xy + xz + yz = 9\). If the sum of the three products \(xy\), \(xz\) and \(yz\) is 9, then their mean is \(9/3 = 3\).