Warm-Up!
1. If the result is 35 when you double the number and add seven, we will start by subtracting 7 from 35 to get 28. So, our number is half of 28, which is 14. To solve algebraically, we first let \( n \) represent our unknown. Then we have \( 2n + 7 = 35 \). Subtracting 7 from each side yields \( 2n = 28 \). Then we divide each side by 2 to get \( n = 14 \).

2. We are told that \( x = y + 3 \) and \( y = z - 5 \), which can be rewritten as \( y + 5 = z \). We are asked to determine the value of \( z - x \). Substituting we get \( (y + 5) - (y + 3) = y + 5 - y - 3 = 5 - 3 = 2 \).

3. In the given expression, \( x \left( \frac{2y + 3}{x} - \frac{2y}{x} \right) \), notice that the expression in parentheses is being multiplied by \( x \), which is the denominator of the two fractions. The \( x \)s cancel, \( x \left( \frac{2y + 3}{x} - \frac{2y}{x} \right) \), and the result is \( 2y + 3 - 2y = 3 \).

4. We are told that the television costs $299 and the older sibling will pay $45 more than the younger sibling. That means that the other \( 299 - 45 = 254 \) dollars will be split equally between the two siblings. Therefore, the younger sibling will pay \( 254 ÷ 2 = 127 \) dollars.

5. From the information given, we can write the following two equations, where \( x \) represents the weight of Tweedledee and \( y \) is the weight of Tweedledum: \( x + 2y = 361 \) and \( 2x + y = 362 \). Adding the two equations we get \( 3x + 3y = 723 \). Dividing each side by 3 we see that the sum of their weights is \( x + y = 241 \) pounds.

The Problem is solved in the MATHCOUNTS Mini.

Follow-up Problems
6. If we let \( n \) represent the numerator and \( d \) represent the denominator then the fraction of which Terry is thinking can be written as \( n/d \). From the second sentence we see that \( (n + 3)/d = 1 \) → \( d = n + 3 \) → \( n = d - 3 \). From the third sentence we see that \( n/(d - 7) = 2 \) → \( n = 2(d - 7) \) → \( n = 2d - 14 \). We can set these two expressions equal to each other to get \( d - 3 = 2d - 14 \) → \( d = 11 \). That means \( n = 11 - 3 = 8 \), and Terry’s fraction is \( 8/11 \).

7. This problem can be solved several ways. First let’s solve it algebraically. We are told that Douglas’ favorite number is a positive two-digit integer; let’s call it \( AB \) where \( A \) is the tens digit and \( B \) is the units digit. That means that the value of his favorite number is \( 10A + B \). Then a new number is created, \( AB7 \), where \( A \) now is the hundreds digit, \( B \) now is the tens digit and 7 is the units digit. The value of the new number is \( 100A + 10B + 7 \). Finally, we are told that the new number is 385 more than Douglas’ favorite number. So we have \( 100A + 10B + 7 = 10A + B + 385 \). Subtracting 10A, B and 7 from both sides yields \( 90A + 9B = 378 \). Dividing both sides by 9 gives us \( 10A + B = 42 \). This is Doug’s favorite number.
We could also have solved the problem logically by setting up the vertical addition problem:

\[
\begin{array}{c}
3 & 8 & 5 \\
+ & A & B \\
\hline
A & B & 7 \\
\end{array}
\]

Notice that \(5 + B = 7\), so \(B\) must equal 2. We can then substitute 2 for \(B\) in the problem to get:

\[
\begin{array}{c}
3 & 8 & 5 \\
+ & A & 2 \\
\hline
A & 2 & 7 \\
\end{array}
\]

The only integer from 1 to 9 that yields a units digit of 2 when added to 8 is 4. It follows that:

\[
\begin{array}{c}
3 & 8 & 5 \\
+ & 4 & 2 \\
\hline
4 & 2 & 7 \\
\end{array}
\]

Thus, Douglas’ favorite number is 42.

8. Let \(p\) represent the number of pit bulls, \(c\) is the number of chihuahuas and \(m\) is the number of mutts. The second sentence of the problem yields the following equations, where \(A\) is the total number of dogs: \(p = A - 23\), \(c = A - 17\), \(m = A - 28\) and \(A = p + c + m\). If we add the first three equations we get \(p + c + m = 3A - 68\). Substituting, we get \(A = 3A - 68\). We now solve to determine that the total number of dogs at the pound is \(2A = 68 \rightarrow A = 34\) dogs.

9. If we add the four equations given, the result is \(3r + 3y + 3b + 3g = 195\). We can divide both sides of the equation by 3 to get \(r + y + b + g = 65\). We can now calculate that there are \(r = 65 - 45 = 20\) red marbles. Likewise, there are \(y = 65 - 45 = 20\) yellow marbles and \(b = 65 - 45 = 20\) blue marbles. There are \(g = 65 - 60 = 5\) green marbles.