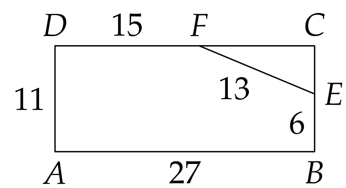
Activity Sheet for the May, 2011, MATHCOUNTS Mini

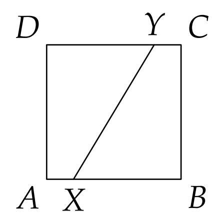
Warm-Up!

Try these problems before watching the lesson.

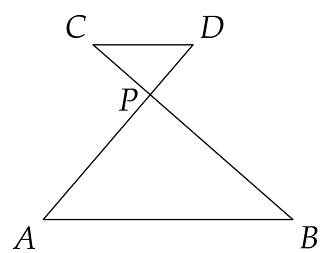
1. A triangular corner region is sliced off from a rectangular region as shown on the right. What is the area of the pentagonal region $ABEFD$ that remains?
Source: MATHCOUNTS



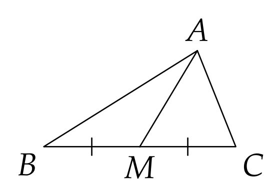
2. $ABCD$ is a square with side length 10. Point X is on side $\overline{AB}$ such that $AX = 2$. A line through X and the center of the square intersects side $\overline{CD}$ at point Y. Find DY and the area of $AXYD$.



3. In the diagram on the right, $\overline{AB}$ and $\overline{CD}$ are parallel, and lines $\overline{AD}$ and $\overline{BC}$ intersect at P. If $AB = 20$, $CD = 8$, and $BC = 14$, then what is CP?



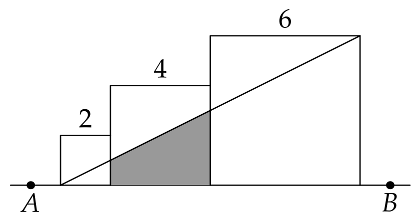
4. A median of a triangle is a segment that connects a vertex of the triangle to the midpoint of the opposite side. For example, in the diagram on the right, $\overline{AM}$ is a median of $\triangle ABC$. Explain why a median of a triangle divides the triangle into two pieces with equal area.



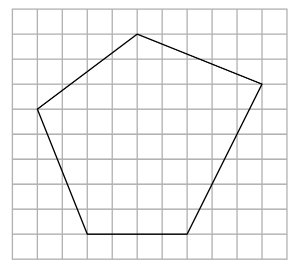
5. Trapezoid $PQRS$ has $\overline{PQ} \parallel \overline{RS}$. If $PQ = 3$ and $RS = 9$, and the area of the trapezoid is 24, then what is the area of $\triangle PQR$?

The Problem

First Problem: Three coplanar squares with sides of lengths two, four, and six units, respectively, are arranged side-by-side, as shown so that one side of each square lies on line AB and a segment connects the bottom left corner of the smallest square to the upper right corner of the largest square. What is the area of the shaded quadrilateral?



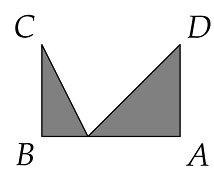
Second Problem: The vertices of a convex pentagon are $(-1,-1)$, $(-3,4)$, $(1,7)$, $(6,5)$ and $(3,-1)$. What is the area of the pentagon?



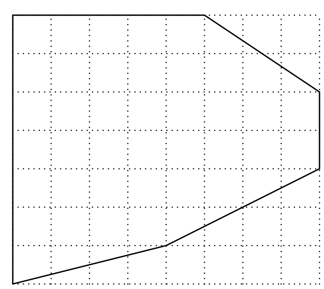
Third Problem: Trapezoid $ABCD$ has base $AB = 20$ units and base $CD = 30$ units. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at X. If the area of trapezoid $ABCD$ is 300 square units, what is the area of triangle BXC?

Follow-up Problems

6. In the figure on the right, $AB = 12$ cm and $BC = AD = 8$ cm. We also have $\overline{BC} \perp \overline{AB}$ and $\overline{DA} \perp \overline{AB}$. How many square centimeters are shaded?



7. Tina wants to carpet a room that has the unusual shape shown on the right with solid lines. Each dotted square in the diagram has side length 5 feet. What is the area of Tina's carpet?

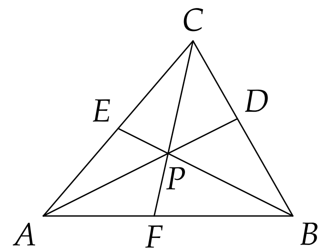


8. Suppose point P is on side $\overline{AB}$ of $\triangle ABC$ such that we have $AP : PB = 1 : 2$. If the area of $\triangle CAP$ is 40 square units, then what is the area of $\triangle ABC$?

9. In quadrilateral $WXYZ$, sides $\overline{WX}$ and $\overline{YZ}$ are both perpendicular to $\overline{XY}$. If $XY = WX + YZ = 8$ units, then what are the possible values of the area of $WXYZ$?

10. Explain how Harvey's slick solution at the end of the video works.

Further Exploration



A segment from the vertex of a triangle to a point on the opposite side of the triangle is called a cevian. In the diagram at the right, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are cevians of $\triangle ABC$. Ceva's Theorem says that cevians $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ all pass through a common point if and only if

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1.$$

Using the following steps, we can prove part of this theorem. Namely, we can show that if the three cevians $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ meet at a point P, then $\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$.

In all of the following parts, let $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ meet at a point P.

11. Show that $\dfrac{\text{Area of } \triangle APF}{\text{Area of } \triangle BPF} = \dfrac{AF}{FB}$.
12. Show that $\dfrac{\text{Area of } \triangle ACF}{\text{Area of } \triangle BCF} = \dfrac{AF}{FB}$.
13. Show that $\dfrac{\text{Area of } \triangle ACP}{\text{Area of } \triangle BCP} = \dfrac{AF}{FB}$.
14. Show that $\dfrac{AF}{FB} \cdot \dfrac{BD}{DC} \cdot \dfrac{CE}{EA} = 1$.

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).