



Activity Sheet for the December, 2010, MATHCOUNTS Mini



Try these problems before watching the lesson.

- 1. Sam flips a coin 7 times. How many different 7-flip sequences are possible?
- 2. A set is a collection of different objects, such as numbers or people. A subset of a set is a collection of different objects that are in the original set. So, subsets of the set $\{1, 2, 3, 4, 5\}$ include $\{1, 2\}$ and $\{1, 4, 5\}$. We also say that the "empty set," which has no elements at all, is a subset of every set. How many subsets does the set {Michelle, Laura, Hillary, Barbara, Nancy} have?
- 3. (a) How many three-digit numbers are multiples of 7?
 - (b) How many three-digit numbers are *not* multiples of 7?
- 4. How many five-digit numbers have at least one zero?
- 5. How many numbers between 150 and 350 are not perfect squares?



How many subsets of the set {M, A, T, H, C, O, U, R, S, E} contain at least one vowel?



Follow-up Problems

- 6. How many subsets of the set {M, A, T, H, C, O, U, R, S, E} contain at least two letters?
- 7. How many three-digit numbers have exactly one zero?
- 8. How many five-letter "words" with at least one vowel can be constructed from the letters A, C, E, G? (Letters may be used more than once in the word.)
- 9. How many four-digit numbers are there such that the thousands digit is double the units digit?
- 10. There are 9 children in Mrs. T's English class, 5 girls and 4 boys. In how many ways can Mrs. T seat them in a row of 9 chairs such that at least 2 girls are next to each other?
- 11. How many sequences of five letters are such that no two adjacent letters are the same?
- 12. What percentage of four-digit numbers have a repeated digit? Express your answer as a decimal to the nearest tenth.

Further Exploration

The following problem is appropriate for students who are familiar with binomial coefficients.

13. Explain how to use counting the subsets of a set with n items to prove that

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^{n}.$$
Wow Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).