Probability Meeting (Probability)

Topic
This meeting’s topic is probability. Students will progress through problems that start with simple comparisons of “successful outcomes” to “possible outcomes” and finish with dependent and independent events.

Materials Needed
♦ Copies of the Probability problem set (Problems and answers can be viewed below. Complete solutions and a more student-friendly version of the problems— with pictures and larger font—are available for download from www.mathcounts.org on the MCP Members Only page of the Club Program section.)
♦ Calculators
♦ OPTIONAL
♦ Dice

Meeting Plan
Probability is often a topic that is difficult for students simply because they often do not have a significant amount of time dedicated to working with probability during regular math class instruction. The following problems cover the spectrum of very easy to rather difficult.

It is recommended that the teacher explain the phrase, “Express your answer as a common fraction.” MATHCOUNTS uses this phrase to remind students that any answer given in the form of a fraction must be reduced to lowest terms. In MATHCOUNTS competitions, only answers given in lowest terms are accepted as correct. However, for the purposes of the club meetings, students arriving at an equivalent answer should certainly recognize that they have successfully figured out the problem even though the form of their answer does not match that in the answer key.

1. Nine cards are numbered 1 through 9. What is the probability of selecting a card with a number greater than four or an even number? Express your answer as a common fraction. 2006 School Competition Countdown Round #46

2. At an awards ceremony 280 personalized certificates were given out in the categories and quantities shown here. Julie earned certificates in both fine arts and mathematics. The principal grabbed a certificate out of the stack at random after all of the science and social studies certificates were given out. If no other certificates had been given out, what is the probability that he pulled out one of Julie’s certificates? Express your answer as a common fraction. 2005-2006 School Handbook Warm-Up 3-7

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3. What is the probability that a positive integer less than or equal to 24 is a factor of 24? Express your answer as a common fraction. 2006 School Competition Sprint Round #10

4. There are three times the number of orange fish as blue fish in a tank at the pet store, and there are no other fish. If Kaya randomly pulls out one fish from the tank, what is the probability that it is orange? Express your answer as a percent. 2005-2006 School Handbook Warm-Up 8-2
5. In a school of 250 students, everyone takes one English class and one history class each year. Today, 15 total students were absent from their English class, and 10 total students were absent from their history class. Five of the students were absent from both classes. If a student is chosen at random from this school, what is the probability that s/he was not absent from either class? Express your answer as a percent. 2004-2005 School Handbook Warm-Up 7-6

6. A point having whole-number coordinates is selected at random from the line $20x + y = 100$. What is the probability that the sum of the coordinates is less than 30? Express your answer as a common fraction. 2002-2003 School Handbook Warm-Up 9-8

7. A jar contains 28 marbles. Half of the marbles are red. Half of the non-red marbles are white and the rest are blue. Todd chose a white marble at random and kept it. What is the probability that when Hosea now draws a marble it will also be white? Express your answer as a common fraction. 2006 School Competition Team Round #9

8. Tara rolls three standard dice once. What is the probability that the sum of the numbers rolled will be three or more? Express your answer as a percent. 2005-2006 School Handbook Warm-Up 1-10

9. If a standard six-sided die is rolled twice, what is the probability that the result of the second roll is not less than the result of the first roll. Express your answer as a common fraction. 2007-2008 School Handbook Warm-Up 3-9

10. Anderson will roll two standard six-sided dice once. What is the probability that the two numbers rolled will be the same? Express your answer as a common fraction. 2007-2008 School Handbook Warm-Up 7-4

11. Russell rolls a pair of dice with his eyes closed. His sister tells him that the sum of the dice is greater than or equal to nine. Knowing this information, Russell calculates the probability that he rolled doubles. What is that probability expressed as a common fraction? 2005-2006 School Handbook Warm-Up 13-7

12. Twenty tiles are numbered 1 through 20 and are placed into box A. Twenty other tiles numbered 11 through 30 are placed into box B. One tile is randomly drawn from each box. What is the probability that the tile from box A is less than 15 and the tile from box B is even or greater than 25? Express your answer as a common fraction. 2006 School Competition Team Round #4

13. Bag A contains four yellow marbles, two green marbles and three blue marbles. Bag B contains four blue marbles and four green marbles. If Anthony choose one marble from each bag, what is the probability that he will select one blue marble and one yellow marble? Express your answer as a common fraction. 2005-2006 School Handbook Warm-Up 10-10

14. Three digits are randomly selected without replacement from two 2s, three 6s and one 9. What is the probability that the sum of the three digits is a multiple of 2? Express your answer as a common fraction. 2005-2006 School Handbook Warm-Up 14-8

Answers: 7/9; 2/213; 1/3; 75%; 92%; 1/3; 2/9; 100%; 7/12; 1/6; 1/5; 21/50; 2/9; 1/2

Possible Next Steps (or Prior Steps)
Problems #8-11 each have situations that use dice. Perhaps before the activity, the club members could be split into four groups. Each group would be given one of the four scenarios in Problems #8-11 and would be asked to simulate the situation. For example, to simulate Problem #9, one group’s members could take turns rolling a die twice. For each person’s pair of rolls, the group will record whether “the second roll was not less than the first roll.” After performing about 25 pairs of rolls, the ratio of “the number of pairs of rolls where the second roll was not less than the first roll” to “total pairs of rolls” could be calculated (experimental probability).

After the experimental probability is calculated for each of the four situations, students could then compare those ratios to the theoretical probability of each of the events (the answers to Problems #8-11).
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2. At an awards ceremony 280 personalized certificates were given out in the categories and quantities shown here. Julie earned certificates in both fine arts and mathematics. The principal grabbed a certificate out of the stack at random after all of the science and social studies certificates were given out. If no other certificates had been given out, what is the probability that he pulled out one of Julie’s certificates? Express your answer as a common fraction.

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14. ___________ Three digits are randomly selected without replacement from two 2s, three 6s and one 9. What is the probability that the sum of the three digits is a multiple of 2? Express your answer as a common fraction.
Problem 1. There are 9 cards. The ones that meet the requirement are 2, 4, 5, 6, 7, 8 and 9. That is \( \frac{7}{9} \).

Problem 2. Since the 38 science certificates and the 29 social studies certificates were already given out, we can calculate that there were \( 280 - 38 - 29 = 213 \) certificates left. Two of those 213 certificates are to be awarded to Julie and have her name on them. Thus, the probability that the principal pulled out one of Julie’s certificates is \( \frac{2}{213} \).

Problem 3. The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24. There are 24 positive integers less than or equal to 24 and 8 of them are factors. The probability is then \( \frac{8}{24} = \frac{1}{3} \).

Problem 4. If there are three times as many orange fish as blue fish in the tank, then three out of every four fish are orange. The probability that Kaya randomly pulls out an orange fish is \( \frac{3}{4} \) or 75%.

Problem 5. If we subtract both 15 and 10 from 250 (the total number of students), we will have deducted twice the five students who were absent from both English and history classes. Adding these 5 back to the 225 figure, we get the correct figure of 230 students who were present in both classes today. The probability that a randomly chosen student was present in both classes is then \( \frac{230}{250} = \frac{23}{25} = \frac{92}{100} = 92\% \).

Problem 6. The equation of the line \( 20x + y = 100 \) can be converted to slope/intercept form as \( y = -20x + 100 \). The slope is negative and the line passes briefly through the first quadrant where both coordinates are positive. The \( y \)-intercept is 100. Since zero is a whole number, this point \((0, 100)\) is one possibility. The others are \((1, 80), (2, 60), (3, 40), (4, 20)\) and \((5, 0)\). Only the last two of the six are such that the sum of the coordinates is less than 30. The probability is thus \( \frac{2}{6} = \frac{1}{3} \).

Problem 7. If the jar contains 28 marbles and half are red, then 14 marbles are red. The rest are split evenly between white and blue. So there are 14 red, 7 white and 7 blue. However, Todd takes out 1 white marble. Now there are a total of 27 marbles. Only 6 of those would give a positive result of Hosea also selecting a white marble. This is a probability of \( \frac{6}{27} = \frac{2}{9} \).

Problem 8. The smallest possible sum Tara can get when she rolls three dice is 3. This means that every one of the \( 6 \times 6 \times 6 = 216 \) possible rolls will have a sum of three or more, and the probability is 100%.

Problem 9. If the first roll is 1, then there are six numbers on the second roll that will not be less. If the first roll is 2, then there are five numbers that will not be less. If the first roll is 3, then there are four, and so on. There are 36 possible outcomes when a die is rolled twice. Of these, \( 6 + 5 + 4 + 3 + 2 + 1 = 21 \) are such that the second roll is not less than the first. The desired probability is thus \( \frac{21}{36} \) or \( \frac{7}{12} \).
Problem 10. If we make a table of all of the possible rolls of two dice, we see that there are 36 possibilities. Only six of these will have the same rolled numbers. Thus, the probability of rolling the same number on each die is $6/36 = 1/6$.

Problem 11. There are four ways to roll a sum of 9: $6 + 3$, $5 + 4$, $4 + 5$ and $3 + 6$. There are three ways to roll a sum of 10: $6 + 4$, $5 + 5$ and $4 + 6$. There are two ways to roll a sum of 11: $6 + 5$ and $5 + 6$. And there is one way to roll a sum of 12: $6 + 6$. Of these 10 possible rolls that are greater than or equal to 9, only two ways are doubles, so the probability that Russell rolled doubles is $2/10$ or $1/5$.

Problem 12. Since each selection is independent of the other, we will need to find the product of each of the individual probabilities. The probability of pulling a number less than 15 from box A is $14/20 = 7/10$. The probability of pulling a number even or greater than 25 from box B is $12/20 = 3/5$. The probability of both of these events happening is $(7/10)(3/5) = 21/50$.

Problem 13. Since there are no yellow marbles in bag B, we have to get the yellow marble from bag A and the blue marble from bag B. Four of the nine marbles in bag A are yellow, so the probability of getting a yellow marble from bag A is $4/9$. Similarly, the probability of getting a blue marble from bag B is $4/8$ or $1/2$. The probability of getting the yellow from bag A and the blue from bag B is the product of these separate probabilities, which is $4/9 \times 4/8 = 2/9$.

Problem 14. If the sum of the three digits selected at random is to be a multiple of 2 (an even number), then we simply cannot select the 9, which is the only odd digit. The probability of not selecting the 9 on the first pick is $5/6$. The probability of not selecting the 9 on the second pick (assuming that it was not selected on the first pick) is $4/5$. The probability of not selecting the 9 on the third pick (assuming it was not selected on the first or second pick) is $3/4$. The probability of these three events happening in succession is $5/6 \times 4/5 \times 3/4 = 1/2$. 

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