Topic
This meeting’s topic is patterns.

Materials Needed
♦ Patterns Stretch on page 38 in Volume II of the 2007-2008 MATHCOUNTS School Handbook
♦ Patterns problem set (available for download from www.mathcounts.org/MCP0708Resources)

Meeting Plan
To start, have students take a look at #2, #5 and #9 of the Patterns Stretch in Volume II of the 2007-2008 MATHCOUNTS School Handbook (shown below). To encourage discussion and teamwork, have students work in groups on the problems. After they have had a chance to do the three problems, ask students to describe how they solved them. (The step-by-step solutions on page 61 in Volume II of the 2007-2008 MATHCOUNTS School Handbook may provide additional solutions that will be fun to discuss with your students.)

2. A pentagon train is made by attaching regular pentagons with 1” sides so that each pentagon, except the two on the ends, is attached to exactly two other pentagons along sides, as shown. How many inches are in the perimeter of a pentagon train made from 85 pentagons?

5. The first three towers in a sequence are shown. The $n^{th}$ tower is formed by stacking $n$ blocks on top of an $n$-by-$n$ square of blocks. How many blocks are in the $99^{th}$ tower?

9. If the pattern shown is continued, what is the sum of the terms in Row 12?

Answers: 257 inches; 9900 blocks; 156

Once these problems have been discussed, we can modify the three questions:
#2: How many inches are in the perimeter of a pentagon train made from 37 pentagons?
#5: How many blocks are in the 37th tower?
#9: What is the sum of the terms in Row 37?
(Note: We chose 37 here, but feel free to let the students pick a number, use the date or year, use your school address, etc.)

Now that your students have a better understanding of patterns, there is a handout you can download from www.mathcounts.org/MCP0708Resources with the patterns problems shown below. These have been pulled from previous MATHCOUNTS materials. Make as many copies of these as you need. Your students can work through them independently or in groups.

1. The first figure in this pattern is a 2 by 2 square, with an area of 4 square units. In the second figure, a congruent square is placed behind the first square such that the midpoints of the left and bottom sides intersect at the midpoints of the top and right sides, respectively, of the first square. In each successive figure, a congruent square is placed behind the preceding, intersecting in the same way. What will be the area, in square units, of the complete region of the fifth figure in the pattern? '06-07 Warm-Up 3-9
2. A chain with two links is 13 cm long. A chain made from three links of the same type is 18 cm long. How many centimeters are in the length of a chain made from 25 such links? '02-'03 Warm-Up 5-4

3. Congruent segments are used to form equilateral triangles in this sequence so that each figure contains one more triangle than the preceding figure. Figure 3 of the sequence is made with seven congruent segments. How many congruent segments must be used to make Figure 25 of the sequence? 2007 Chapter Sprint

4. In this pattern, a new segment with endpoint A is added to the newest figure to create the next figure. If L is the number of line segments and P is the number of endpoints for a particular figure, what is the value of L + P in Figure 10? (The value of L + P in Figure 1 is 3 + 4 = 7.) '06-'07 Warm-Up 16-8

5. A total of 180 marbles (gray, white, black) are placed in a line. The first five are gray, followed by four white, followed by three black, followed by five gray, followed by four white, followed by three black, ... . If this pattern continues, what is the color of the 158th marble in this line? '07-'08 Warm-Up 3-6

6. Yusuf is creating a border pattern for a wall. Using the shape shown to the right, the first five shapes of the pattern are given below. Each time, Yusuf moves the shape to the right and rotates it 90 degrees clockwise about point A. Draw the 97th shape of the pattern. '04-'05 Transformation Stretch - 2

7. A set of magnetic strips and balls are connected in an alternating fashion, as shown. The total length of the first four items in the chain is 14 cm, and the total length of the first seven items is 25 cm. What is the total length, in cm, of the first 25 items in the chain? '04-'05 Warm-Up 5-1

8. Ann starts counting the letters of the alphabet beginning with A. When she gets to Z, she goes backwards from Y to A and then reverses again going from B to Z. If she continues this process, what is the 2005th letter that she will count? '04-'05 Warm-Up 10-9

9. The first 20 numbers of an arrangement are shown. What would be the value of the 40th number if the arrangement were continued? '03-'04 Warm-Up 9-4

<table>
<thead>
<tr>
<th>Row 1</th>
<th>Row 2</th>
<th>Row 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2, 2</td>
<td>4, 4, 4, 4</td>
<td>6, 6, 6, 6, 6</td>
</tr>
<tr>
<td>2, 5</td>
<td>7, 5</td>
<td>8, 8, 8, 8, 8</td>
</tr>
</tbody>
</table>

10. In this array of numbers, the first number in each row is 2 and the last number in each row is 5. Each of the other entries is the sum of the two numbers nearest it in the row directly above it. What is the sum of all of the numbers in row 10? 2004 Chapter Team Round

Answers: 16 square units; 128 centimeters; 51 segments; 25; gray; same orientation as the first figure in the pattern; 88 cm; E; 12; 3584
Possible Next Steps
If students finish early, they can go back and come up with algebraic expressions that represent each of the patterns. For instance, for the pentagon trains, they could say that the perimeter always is $4 + 4 + 3(n - 2)$ inches or $3n + 2$ inches for $n$ pentagons. For the tower problem, we were really given that we always will need $(n)(n) + n = n^2 + n$ blocks for the $n^{\text{th}}$ tower. For the sum of the terms in Row $n$, we could say that this always will be $n((2 + 2n) ÷ 2)$ or $n(n + 1)$. These are not the only forms of these answers.

Another idea is to have your students try to stump each other with pattern problems that they create. The ones we have focused on are pretty visual, but there are more examples of patterns in the rest of the Patterns Stretch problems that we encourage you to use.

We would love to see some of the problems your students write. If you feel any are particularly great, please e-mail them to info@mathcounts.org with the subject line “MATHCOUNTS Club Program.” Perhaps we can use them in future school handbooks... be sure to give us the name of the author so we can give him/her proper credit!
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