Game and Puzzle Month (Nov.) Meeting
(Multiple Topics)

Topic
There are a variety of math topics covered in the problems used for this meeting.

Materials Needed
♦ Copies of the Game and Puzzle Month problem set (Problems and answers can be viewed here, but a more student-friendly version in larger font is available for download from www.mathcounts.org on the MCP Members Only page of the Club Program section.)
♦ Prizes for winners of games you play during the meeting
♦ Calculators

Meeting Plan
November is Game and Puzzle Month, so we have assembled some of our favorite MATHCOUNTS problems that relate to games and puzzles. They are in order of difficulty, with #11–#14 being pretty challenging. Note: You may wish to start the meeting using the “Possible Next Step” for #14 that is described at the end of this activity.

1. Three black chips have been placed on the game board shown. What is the number in the square where a fourth chip should be placed so that it shares neither a row nor a column with any of the existing chips? 2006 Chapter Competition, Sprint #2

2. This addition square is a 3-by-3 grid of 9 numbers such that, in each row from left to right, and in each column from top to bottom, the first number plus the second number equals the third number. When this addition square is filled in completely, what is the largest possible sum of all 9 numbers? 2006–2007 School Handbook, Workout 2-8

3. The puzzle pictured is a prism consisting of three identical cubes which may be twisted until each vertical face of the prism is a single color. What color is the cube’s face marked “?”? 2001–2002 School Handbook, Warm-Up 16-3

4. In the maze to the right, you may move only vertically or horizontally. Each square you move to must have a fraction that is a value less than the fraction of the square you are currently in. If you start at the indicated square, what fraction is in the last square of your path on the bottom row? 2005–2006 School Handbook, Warm-Up 15-4

5. When going through this maze, Mikyong may not backtrack. Each time she hits the edge of the grid or runs into a dark blocker, she must turn. If Mikyong comes to a dead end (no options to move), the game is over. How many paths result in a dead end? 2006–2007 School Handbook, Warm-Up 6-9

6. In this magic square, the product of the three numbers in each row is 4096, and the product of the three numbers in each column is also 4096. When the magic square is filled in, what number is in the shaded region? 2005–2006 School Handbook, Workout 2-8
7. In the maze to the right, a move is either horizontal or vertical in a single direction. Each move ends when you reach a barrier or the edge of the grid, where you must turn 90 degrees in either direction and then start your next move. You must always remain within the grid. The thicker lines indicate barriers or edges. The 10 moves of one possible route from square A to square B are shown with arrows. What is the least number of moves needed for a route from square A to square B? 2006 School Competition, Sprint Round #4

8. In a particular dice game, a player rolls two dice. The player can choose either the sum or the product of the numbers rolled for her score for that turn. To win the game, a player must get a total score of exactly 102 points from the sum of her scores. What is the least possible number of turns needed to win the game? 2005-2006 School Handbook, Warm-Up 3-9

9. In how many ways can this figure be colored if the five regions must be colored with either red, white or blue and no two bordering regions can be the same color? 2005–2006 School Handbook, Warm-Up 16-1

10. Marcus and Al are playing Rock/Paper/Scissors. The first person to win 10 times does not have to do the dishes. The score is now 9 to 7 in favor of Marcus. If ties are ignored and not counted, what is the probability Marcus will not have to do the dishes? Express your answer as a common fraction. 2005–2006 School Handbook, Probability Stretch #6

11. Each of the nine dots in this figure is to be colored red, white or blue. No two dots connected by a segment (with no other dots between) may be the same color. How many ways are there to color the dots of this figure? 2006–2007 School Handbook, Workout 5-7

12. In the number puzzle to the right, each of the eight non-shaded unit squares contains one digit. What is the answer to 1-Across? (Note: This works like a crossword puzzle in that the digit placed in the unit square with the 5 in the upper left corner is the first digit of the answer to 5-Across and the third digit of the answer to 1-Down. No digit is placed in the shaded square.) 2005–2006 School Handbook, Workout 5-7

Across
1. A prime number
3. A perfect square
5. A perfect fourth power

Down
1. A multiple of 9
2. A perfect cube
4. A perfect square

13. Two standard six-faced dice are rolled. Cara scores $x$ points if the sum of the numbers rolled is greater than or equal to their product; otherwise, Jeremy scores one point. What should be the value of $x$ to make the game fair? 2002–2003 School Handbook, Workout 8-8

14. Ron and Ameka are playing a game in which each player can take 1, 2 or 3 coins on each turn. The game begins with 17 coins in a pile, and the player to take the last coin from the pile wins. If Ameka goes first, how many coins should she take to guarantee that she will win? 2003–2004 School Handbook, Patterns Stretch #9

Answers: 5; 100; Red; 7/3; 9 paths; 8; 5 moves; 3 turns; 12 ways; 7/8; 54 ways; 61; 2; 1 coin

Possible Next Steps

Students can now create number puzzles like that found in #12. The creation of these is often more difficult than it may first appear. Students need to be able to correctly describe numbers and ensure that there is just one possible solution for the puzzle — if that’s a requirement you make. Please share these with us if you get some great ones from your students.

Problem #14 is a great game to play with your students. The frustration of you always winning will certainly spark their curiosity to figure out the trick! (No matter what number of coins you start with, always take enough to leave a multiple of 4 after your turn. Then, no matter how many the next person takes, you can again take enough to leave a multiple of four. This will ensure that there are four left on your opponent’s last turn, they can’t take all four, and you will be able to take what is left. Always go first if the starting number is not a multiple of 4, and you’ll win. If you can’t go first, hope your opponent doesn’t know the trick, and you can usually get it back on track in your favor.)
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Copyright MATHCOUNTS, Inc. 2008. MATHCOUNTS Club Resource Guide Problem Set
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<tbody>
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**Answers to these problems are on page 32 of the 2008-2009 Club Resource Guide.**
**Problem 1.** Rows 1, 3 and 4 have chips. Therefore, one must be in row 2. Columns 1, 3 and 4 have chips. Therefore, one must be in column 2. The number in row 2, column 2 is 5.

**Problem 2.** Students can start with some Guess & Check and see that no matter what actual numbers they choose, as long as they meet the restrictions, the nine numbers will always add to 100. This is because the first two numbers in the last column – let's call them $x$ and $y$ for later – add to 25, the first two numbers in the last row add to 25 and there is the 25 that is already given. This gets us to 75. Notice that the first two numbers in the top row have to equal $x$, and the first two numbers in second row have to equal $y$, so all four of those numbers equal $x + y = 25$. They give us our other 25 to get to 100.

**Problem 3.** From the second cube, we can tell that Green must be to the left of White in the bottom cube. What is to the right of White? We're missing Red and Blue, and from the top cube we can see that Red is to the left of Blue, so **Red** belongs on the face marked “?”.

**Problem 4.** If we make our first move to the left from $24/5$ to $32/8$, we come to a dead end since there is no smaller fraction for the next step. Let's move from $24/5$ to $23/6$. From $23/6$, we can only move down to $11/3$. From $11/3$, we can either move down to $24/8$ or right to $18/5$. If we move right to $18/5$, we will have to move down to $20/6$ and then left to $24/8$, so the two paths come back together at $24/8$. From $24/8$, we can move only left to $12/5$ and then down to $7/3$. The fraction in the last box on the bottom row is $7/3$.

**Problem 5.** There are 9 possible dead-end paths in the maze:

![Maze Diagram]

**Problem 6.** We can find the value of each empty cell by dividing 4096 by each of the known numbers in each row or column. We get 128 and 4 on the left and 256 and 2 on the bottom. Then we find that the value of the shaded region must be 8. It is interesting to note that each of the numbers is a power of 2, which means that we could do a simpler version of this magic square by adding exponents. The exponents would be 7, 0, 5 on the top row; 2, 4, 6 on the middle row; and 3, 8, 1 on the bottom row. The sum of each row, column and the two diagonals is 12, and $2^{12} = 4096$.

**Problem 7.** Going left from A, then down, then left, then down, then left, we get from A to B in 5 moves.

**Problem 8.** A player could get 102 points in 3 turns if she rolled 6 & 6 twice and a 5 & 6 once. The player would choose the product for each roll, which would amount to $36 + 36 + 30 = 102$ points.

**Problem 9.** The square in the upper left corner can be any of the three colors. The two adjacent regions must be colored with the other two colors. This can be done in two ways. The triangle on the lower right must be the same color as the square in the upper left. Finally, the region in the upper right can be either of the colors not used in the adjacent region. Thus, there are $3 \times 2 \times 1 \times 2 = 12$ ways to color the figure.
Problem 10. They have played 16 duals, and the score is 9 to 7 in favor of Marcus. The game of Rock/Paper/Scissors is a fair game, which means that Marcus and Al each have a 1/2 probability of winning each dual. The only way Marcus could lose from the position of 9 to 7 is if Al wins the next three duals in a row. The probability of Al winning three in a row is \( \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} \), so Marcus must have a \( 1 - \frac{1}{8} = \frac{7}{8} \) probability of winning.

Problem 11. There are six ways we can color the first triangle on the left. With that in mind, let’s go ahead and set that first triangle as Red on top, Blue on lower left and White on lower right. With that triangle set, you will see that there are only three ways to color the vertices of the second (middle) triangle. Placing a Blue on top limits the bottom vertices. However, placing a White on top leaves two options for the bottom vertices. Then, with that second triangle set, there will be three ways to color the vertices of the last triangle. Therefore, there are \( 6 \times 3 \times 3 = 54 \) ways to color the vertices of the figure.

Problem 12. We want to start with the most restrictive clue, which is the clue for 5-Across. There are only two three-digit perfect fourth powers: \( 4^4 = 256 \) and \( 5^4 = 625 \). Next, we might consider the clue for 2-Down. There are only five three-digit perfect cubes: \( 5^3 = 125, 6^3 = 216, 7^3 = 343, 8^3 = 512 \) and \( 9^3 = 729 \). Since the units digit of the perfect cube will be the tens digit of the perfect fourth power, we can determine that there are two possibilities: perfect fourth power 625 with perfect cube 512 or perfect fourth power 256 with perfect cube 125. Noting the clue for 1-Across, we can eliminate the first of these possibilities since no two-digit prime ends in a 5. We now know that the prime number for 1-Across must end in a 1. The possible primes are 31, 41, 61 and 71. We also know that the perfect square for 4-Down must be either \( 4^2 = 16 \) or \( 6^2 = 36 \). The clue for 3-Across suggests that we look for a three-digit perfect square with a 2 in the tens place and either a 1 or a 3 in the ones place. Since no square number ends in a 3, we know that 4-Down must be 16. Then the only possible number for 3-Across is \( 11^2 = 121 \). Now to make sure the number for 1-Down is a multiple of 9, we need a 6 in the hundreds place so the sum of the digits is a multiple of 9, or 9. This means the number for 1-Across must be \( 61 \).

Problem 13. Making two tables (one to show the possible sums and one to show the possible products when two standard dice are rolled) will be helpful. You will then see that the sum is greater 12 times out of 36, and the product is greater 24 times out of 36. Since the ratio of Greater Sums to Greater Products is 1:2, Cara should get 2 points for every one point Jeremy scores. The value of \( x \) should equal 2.

Problem 14. The person who goes first can control the game entirely. Ameka should take 1 coin, leaving Ron with 16 coins, which is a multiple of 4. No matter how many coins Ron takes on his next turn, Ameka can again leave him with a multiple of 4 coins. If he takes one, she takes three; if he takes two, she takes two; and if he takes three, she takes one. Eventually, there will be just four coins left on Ron’s turn, and Ron is forced to leave Ameka with one, two or three coins.