2004-2005: What are the colors of the caps in the bottom row?

The poster uses caps that can't be rotated. Therefore, we may not rotate the pieces here with the circles. Our line of symmetry is the key to this problem. We know that any cap can go into a square that the line of symmetry goes through because it must only "match" with itself. However, because of the line of symmetry, we know that a yel-

Figure 1

low cap must be in the bottom left corner, a blue cap must be right above it and a red cap must be in the first square of the second row. Of the pieces that we are given to use, only two of them have yellow caps. One of them must be positioned so that a yellow cap is in the bottom left corner. Let's take the piece with three vertical hats (yellow at bottom) and place it in the first column leaving the top square of the column empty. This
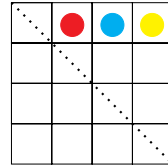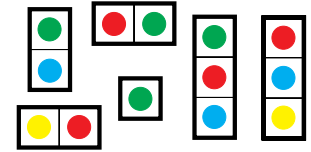

Figure 2

actually perfectly matches the symmetry of the three original hats that were already placed. We now have the matching red, blue and yellow hats in place. The top left corner square is open now, and we can see that only the piece with the single green hat fits in the spot (Figure 2). Here's where this falls apart, though. Notice that there are only two blue hats in our remaining pieces. Either they are both in squares along the line of symmetry, or neither of them are on the line of symmetry and they are opposite/symmetric to each other. The only two

Figure 3

possibilities are shown in Figure 3. Notice that there are not possible fits for the remaining two pieces in either of these cases. This tells us that our initial placing of that piece with the yellow hat was the wrong choice. It forced us into this dead end.
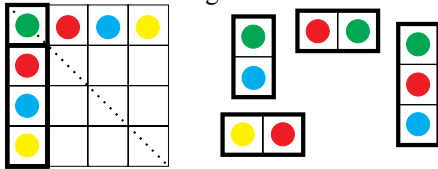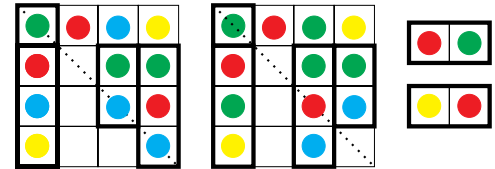
We need to place the other piece with the yellow hat so that the yellow hat is in the bottom left corner (Figure 4). Now there is only one piece that allows us to have symmetry with the new red hat we just added.
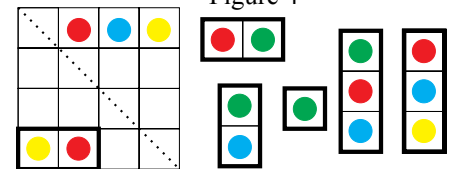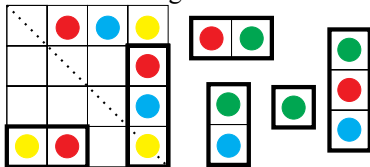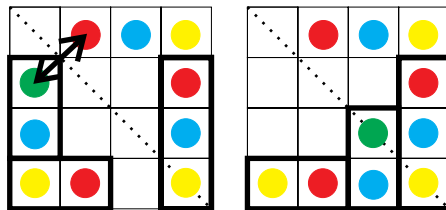

Figure 4

Figure 5

So we place the piece with three (top red) into the fourth column (Figure 5). Notice that we have two pieces left with blue hats, and two mandatory spots in the figure that must have blue hats to complete the symmetry. If we place the piece with two hats (green and blue) in the first column to match the blue hat in the first row, notice that the newly placed green hat doesn't match properly with the original red hat. This tells us that this particular two-hat piece must be placed so that the blue hat is in the bottom row. The green hat is on the line of symmetry and does not have to match up with any other hat (Figure 6). Now the remaining piece with a blue hat fits nicely into the first column (and its red and green hats match up nicely, too). This leaves obvious placements for the final two pieces, and we maintain our symmetry (Figure 7). This poster is easiest to do if students can actually cut out the pieces and physically move them around to fit into the cubby holes! The bottom row of hats is yellow, red, blue and yellow.

Figure 6

Figure 7