1. How many integers between 200 and 300 have the sum of their digits equal to 15?

2. Following the arrows, how many different routes are there from A to D?

3. When this net of six squares is cut out and folded to form a cube, what is the product of the numbers on the four faces adjacent to the one labeled with a “1”?

4. Define \( A \@ B \) to be \( \frac{A}{B} + (A \times B) \). What is the value of \( 20 \@ (4 \@ 2) \)?

5. Five balls are numbered 1 through 5 and placed in a bowl. Josh will randomly choose a ball from the bowl, look at its number and then put it back into the bowl. Then Josh will again randomly choose a ball from the bowl and look at its number. What is the probability that the product of the two numbers will be even and greater than 10? Express your answer as a common fraction.
1. **6** integers  How many integers between 200 and 300 have the sum of their digits equal to 15?

Any integer between 200 and 300 will have a hundreds digit of 2. All of our integers will be of the form 2 _ _. So the remaining two digits will have a sum of $15 - 2 = 13$. There are three pairs of digits that have a sum of 13: 9 & 4, 8 & 5 and 7 & 6. Each pair can be used in two ways - with the larger number being our tens digit or our ones digit. Therefore, there are 6 total integers: 294, 249, 285, 258, 276 and 267.

2. **18** routes  Following the arrows, how many different routes are there from A to D?

There are two general ways to get from A to D; we can go A → B → D or A → C → D. For A → B → D, there are two routes from A to B. Each of these two routes can then be matched with any of the three routes from B to D. This is a total of 6 A → B → D routes. For A → C → D, there are three routes from A to C. Each of these three routes can then be matched with any of the four routes from C to D. This is a total of 12 A → C → D routes. Now we know there are $6 + 12 = 18$ routes from A to D.

3. **144**  When this net of six squares is cut out and folded to form a cube, what is the product of the numbers on the four faces adjacent to the one labeled with a “1”?

When the net of squares is cut out and folded to make a cube, the face with “1” is opposite (and parallel to) the face with “5.” Therefore, the other four faces are adjacent to the one labeled with “1.” The product of the numbers on those other four faces is $2 \times 3 \times 4 \times 6 = 144$. 
4. **202** Define $A @ B$ to be $\frac{A}{B} + (A \times B)$. What is the value of $20 @ (4 @ 2)$?

For the expression $20 @ (4 @ 2)$, we must do what is in the parentheses first. Working with $4 @ 2$, we have $\left(\frac{4}{2}\right) + (4 \times 2) = 2 + 8 = 10$. Now we have $20 @ (4 @ 2) = 20 @ 10$. Using our rule again, we have $\left(\frac{20}{10}\right) + (20 \times 10) = 2 + 200 = 202$.

5. **$\frac{1}{5}$** Five balls are numbered 1 through 5 and placed in a bowl. Josh will randomly choose a ball from the bowl, look at its number and then put it back into the bowl. Then Josh will again randomly choose a ball from the bowl and look at its number. What is the probability that the product of the two numbers will be even and greater than 10? Express your answer as a common fraction.

Josh’s first selection will be either 1, 2, 3, 4 or 5. Because that ball is replaced, his second selection also will be either 1, 2, 3, 4 or 5. Each of the first 5 selections can be matched with each of the second 5 selections for a total of 25 possible selections. How many of these will give us a pair of numbers with an even product greater than 10? Let’s go through our options for each of our 5 possible first selections. First pick the 1 → no second selection will work. First pick the 2 → no second selection will work. First pick the 3 → picking the 4 next will work. First pick the 4 → picking the 3, 4 or 5 next will work. First pick the 5 → picking the 4 next will work. This means 5 of the 25 possible selection processes will work, which is $\frac{5}{25} = \frac{1}{5}$.