Calculators may NOT be used on this challenge.

1. _____ gallons   When a water tank is 30% full, it contains 27 gallons less than when it is 20% empty. How many gallons of water does the tank hold when it is full?

2. ______ mph   Stan drove 300 miles in 5 hours, 20 minutes. Next, he drove 360 miles in 6 hours, 40 minutes. What was Stan’s average speed in miles per hour for the total trip?

3. _______   What is the positive difference between the 2000th term and the 2005th term of the arithmetic sequence −8, −2, 4, 10, ... ?

4. _______   In the game “Cover It Up” two standard six-faced dice are rolled and their sum determined. The player can then cover that number on the game board or any two numbers that have that sum. For example, if you roll a 3 and 5, you can cover the 8, the 1 and 7, the 2 and 6, or the 3 and 5. What is the probability of being able to cover the 9 on the first roll of the two dice? Express your answer as a common fraction.

5. ______ answer keys   A teacher has made eight statements for a True-False test. Three statements are true and five are false. How many distinct answer keys could there be for the test?
Calculators may NOT be used on this challenge.

1. **54 gallons** When a water tank is 30% full, it contains 27 gallons less than when it is 20% empty. How many gallons of water does the tank hold when it is full?

If the tank is 20% empty, it is 80% full. Therefore, 27 gallons represents the difference between 80% full and 30% full, which is 50% of the tank. A full tank, or 100% of the tank, is then 27 gallons × 2 = 54 gallons.

2. **55 mph** Stan drove 300 miles in 5 hours, 20 minutes. Next, he drove 360 miles in 6 hours, 40 minutes. What was Stan’s average speed in miles per hour for the total trip?

In order to find the average speed for an entire trip you must divide the total distance traveled by the total time the trip took. In this case, Stan traveled 300 miles + 360 miles = 660 miles. The total travel time was 5 hours, 20 minutes + 6 hours, 40 minutes = 11 hours, 60 minutes = 12 hours. Thus, Stan’s average speed was 660/12 = 55 mph.

3. **30** What is the positive difference between the 2000th term and the 2005th term of the arithmetic sequence −8, −2, 4, 10, ... ?

Each term differs by 6. To see this simply, look at the second and third terms. 
−2 = −8 + (6 × 1); 4 = −8 + (6 × 2)

So we can express this sequence as the \( n = −8 + 6(n − 1) \).

−8 + (6 × 1999) = −8 + 11,994 = 11,986. This is the 2000th term.

−8 + (6 × 2004) = −8 + 12,024 = 12,016. This is the 2005th term.

12,016 – 11,986 = 30

However, if you do it this way, while it is right, it’s really the long way. Since this is an arithmetic sequence, we know that each two successive terms differ by 6. Two terms that are 5 terms away from each other differ by 6 × 5 = 30.
4. \( \frac{5}{18} \) In the game “Cover It Up” two standard six-faced dice are rolled and their sum determined. The player can then cover that number on the game board or any two numbers that have that sum. For example, if you roll a 3 and 5, you can cover the 8, the 1 and 7, the 2 and 6, or the 3 and 5. What is the probability of being able to cover the 9 on the first roll of the two dice? Express your answer as a common fraction.

To cover the 9 we must have a roll of the dice that is greater than or equal to 9. What values are these? Clearly 9, 10, 11 and 12. 9 can be rolled as (4,5), (5,4), (6,3) and (3,6). 10 can be rolled as (6, 4), (4, 6) and (5, 5). 11 can be rolled as (6, 5) and (5, 6). Finally, 12 can be rolled as (6, 6). This is a total of 4 + 3 + 2 + 1 = 10 possible rolls. There are 6 × 6 = 36 possible rolls. 10/36 = 5/18.

5. 56 answer keys A teacher has made eight statements for a True-False test. Three statements are true and five are false. How many distinct answer keys could there be for the test?

If each of the eight questions on the test were either true or false, there are \(2^8 = 256\) possible answer keys. We know, however, that three statements are true and five are false. This time there are perhaps too many possibilities for an organized list. Perhaps we can take a short cut. Imagine that all eight of the questions are false. We must pick three of them to change to true questions. We can choose from a group of eight in “eight choose three” ways, which is \(\binom{8}{3} = 8!(/5! \times 3!) = (8 \times 7 \times 6)/(3 \times 2 \times 1) = 4 \times 7 \times 2 = 56.\) Thus, there are 56 possible answer keys.