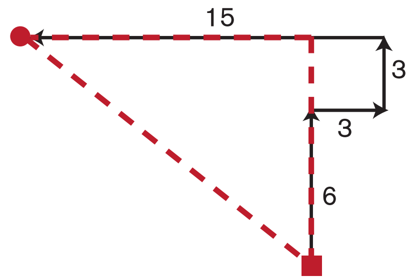
MATHCOUNTS® Minis September 2013 Activity Solutions

Warm-Up!

1. Armond first travels 10 feet at a rate of 1 foot every 10 seconds, then travels 10 feet at a rate of 1 foot every 15 seconds. That means it takes Armond $10 \times 10 = 100$ seconds to travel from the wall to the crumb, and another $10 \times 15 = 150$ seconds to travel back to the wall along the same path carrying the crumb. Therefore, the entire trip takes Armond $100 + 150 = 250$ seconds, which is equivalent to $250 \div 60 = \mathbf{4\frac{1}{6}}$ minutes.

2. Since the trip takes 24 minutes at a rate of 30 mi/h, to make the trip in 12 minutes, which is half the time, the rate needs to be doubled. Therefore, to complete the same trip in 12 minutes, the rate of travel would need to be $30 \times 2 =$ **60** mi/h.



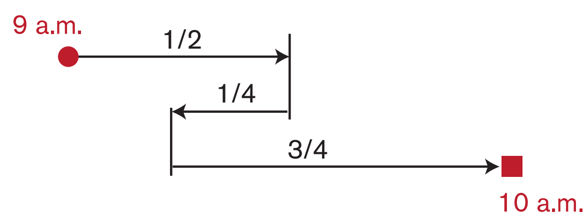
3. As shown, the distance Wally is from his starting point is the length of the hypotenuse of a right triangle with legs of length $6 + 3 = 9$ mi and $15 - 3 = 12$ mi. We can use the Pythagorean Theorem to determine the length of the hypotenuse. We have $9^2 + 12^2 = d^2 \rightarrow 81 + 144 = d^2 \rightarrow d^2 = 225 \rightarrow d =$ **15** mi. You may recognize these side lengths (9 mi, 12 mi and 15 mi) as a multiple of the Pythagorean Triple 3-4-5.

4. We need to determine on what day the frog first reaches the top of the well to escape. Since the frog moves up 5 m each day and down 3 m each night, it follows that after Sunday morning, he begins each day 2 m higher than he began the previous day. He starts 12 m from the top of the well. After moving up 5 m and down 3 m on Sunday, the frog starts Monday 2 m higher than he started on Sunday, which is 10 m from the top of the well. Tuesday morning he will have moved up a total of 4 m, which is 8 m from the top of the well. Wednesday morning he will have moved up a total of 6 m, which is 6 m from the top of the well. Thursday morning he will have moved up a total of 8 m, which means he is only 4 m from the top of the well. Therefore, when he moves up 5 m on **Thursday**, he will be able to exit the well.

The Problem is solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. As shown, Cindy traveled $1/2 + 1/4 + 3/4 = (2 + 1 + 3)/4 = 6/4 = 3/2$ the distance, d, from her house to Jenny's house. We also know that she traveled this distance in 1 hour at a rate of 2 mi/h. Using $d = r \times t$, we can write $[(3/2) \times d] = 2 \times 1 \rightarrow d = 2 \times (2/3) \rightarrow d = 4/3$. So, Cindy and Jenny live **4/3** miles apart.



6. The man has run 2/5 of the way through the tunnel when he hears the approaching train. The amount of time it will take him to run back to the tunnel's entrance is the same amount of time it will take the train to reach the tunnel's entrance. We also know that the amount of time it will take the man to run the remaining 3/5 of the way through the tunnel to its exit is the same amount of time it will take the train to reach the tunnel's entrance and then travel through the tunnel to its exit. But the amount of time it will take the train to reach the tunnel's entrance is the same amount of time it takes the man to run 2/5 of the way through the tunnel. So it follows that the time it will take the man to run the remaining 3/5 of the way through the tunnel is the same as the amount of time required for the man to run 2/5 of the way through the tunnel **plus** the amount of time it will take the train to go through the tunnel and reach the exit. In other words,

time to run 3/5 tunnel = time to run 2/5 tunnel + time for train to go from tunnel entrance to exit

Subtracting the time it will take the man to run 2/5 of the way through the tunnel from both sides of this equation, we see that the time it will take the man to run 1/5 of the way through the tunnel is equivalent to the time it will take the train to go through the tunnel and reach the exit. Based on this, we can conclude that the train's speed is 5 times the man's speed. Therefore, the man must be running at a rate of 60 ÷ 5 = **12** mi/h.

7. Tirunesh and Sally are traveling in opposite directions at a combined rate of 8 + 7 = 15 m/s. At this rate, they will first cover a combined distance of 400 m after 400 ÷ 15 = 80/3 seconds. Since 5 minutes is equivalent to 5 × 60 = 300 seconds, together Tirunesh and Sally will cover this combined distance 300 ÷ (80/3) = 45/4 = 11.25 times. That means Tirunesh and Sally will give each other **11** high fives.

8. Suppose that from noon until 8 p.m. Marco hiked 20 miles. If we plot Marco's distance traveled from his starting point and travel time, it would look similar to Figure 1 below. Figure 2 shows the plots for Marco's first hike to a location 20 mi from his starting point and for Marco's hike from that location, along the same path, back to his starting point. Notice the point of intersection of the two curves. This is the location that Marco visits at the same time on both hikes. These two curves will always intersect at some point if Marco travels along the same path on both hikes.

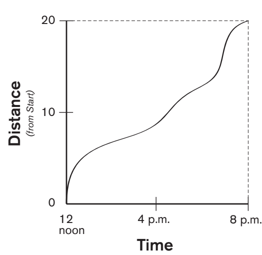


Figure 1

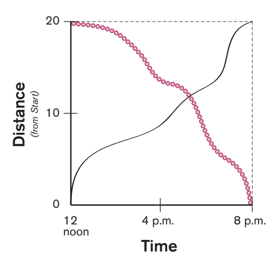


Figure 2

9. If Marco only takes 4 hours for the return trip, the plot would look similar to Figure 3. Again we see that the two curves must intersect as some point, and we can conclude that there must be some location that Marco visits at the same time on both hikes.

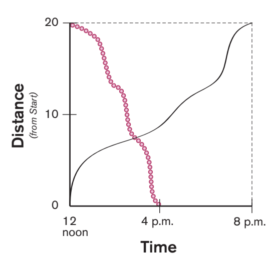


Figure 3