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# MATHCOUNTS®

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2016  
■ State Competition ■  
Team Round  
Problems 1–10

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School \_\_\_\_\_  
Chapter \_\_\_\_\_  
Team \_\_\_\_\_  
Members \_\_\_\_\_, Captain  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

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Total Correct	Scorer's Initials

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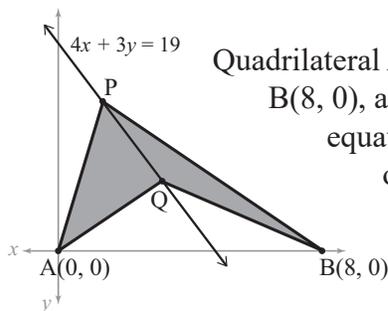
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1. \_\_\_\_\_ mi/h The end of each blade of a ceiling fan is two feet from the center of the fan and makes three full rotations per second. Given that there are 5280 feet in a mile, what is the speed, in miles per hour, of the end of one of the blades? Express your answer as a decimal to the nearest tenth.
2. \_\_\_\_\_ A, B, C, D and E in the decimal representations 0.ABC and 0.DE represent the digits 1, 2, 3, 4 and 5, in some order. What is the least possible absolute difference between the numbers 0.ABC and 0.DE? Express your answer as a decimal to the nearest thousandth.
3. \_\_\_\_\_ percent Approximately 70.8% of Earth's surface is covered with water; the rest is land. It is said that Mars, whose surface is covered entirely with land, has approximately the same amount of land as the earth. Based on this information, what percent of Earth's radius is Mars' radius? Express your answer to the nearest whole number.
4. ( \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ ) Billy wrote a sequence of five numbers on the board, each an integer between 0 and 4, inclusive. Penny then wrote a sequence of five numbers that measured some statistics about Billy's sequence. In particular, Penny first wrote down the number of 0s in Billy's sequence. Then Penny wrote the number of 1s in Billy's sequence, and then the number of 2s, the number of 3s, and finally the number of 4s. It turned out that Penny's sequence was exactly the same as Billy's! What was this sequence? Express your answer as an ordered 5-tuple.
5. \_\_\_\_\_ If  $x$  is a number such that  $3^x + 3^{x+2} = 9^x + 9^{x+2}$ , then what is the value of  $3^x$ ? Express your answer as a common fraction.

6. \_\_\_\_\_ times A standard U.S. dime has a mass of 2.268 grams. A standard U.S. nickel has a mass of 5.000 grams. Diane has a large bag of dimes, and Nick has a large bag of nickels. If both bags of coins have the same total mass, then the total value of Diane's coins is how many times the total value of Nick's coins? Express your answer as a decimal to the nearest tenth.

7. \_\_\_\_\_ For each positive integer  $n$ ,  $a_n = 9n + 2$  and  $b_n = 7n + 3$ . If the values common to both sequences are written as a sequence, the  $n$ th term of that sequence can be expressed as  $pn + q$ . What is the value of  $p - q$ ?

8. \_\_\_\_\_ units<sup>2</sup>



Quadrilateral APBQ, shown here, has vertices A(0, 0) and B(8, 0), and vertices P and Q lie on the line given by the equation  $4x + 3y = 19$ . If  $PQ = 3$  units, what is the area of quadrilateral APBQ? Express your answer as a common fraction.

9. \_\_\_\_\_ arrangements Eight blue and five orange tiles are arranged in an ordered line such that the tile on the left must be blue and every tile must be adjacent to at least one tile of the same color. For example, if an arrangement of four tiles was made, the only possibilities would be BBBB or BBOO. How many different arrangements are possible if all thirteen tiles must be used?

10. \_\_\_\_\_ bags Spring City is replanting the grass around a circular fountain in the center of the city. The fountain's diameter is 10 feet, and the grass extends out from the edge of the fountain 20 feet in every direction. Grass seed is sold in bags that will each cover 300 ft<sup>2</sup> of grass. How many whole bags of grass seed will the city need to purchase?

