

MATHCOUNTS[®] Problem of the Week Archive

Best of 2020! – December 28, 2020

Problems & Solutions

While there were many problems written and solved for MATHCOUNTS in 2020, below is a selection of some of the best of the year!

2019-2020 School Handbook, #42

Janice is 4 inches shorter than David, and David is 2 inches shorter than Evan. If Evan is 13 inches taller than Kathleen and Kathleen is 6 inches shorter than Krysta, by how many inches is Janice taller than Krysta?

To make this situation less abstract, let's suppose Janice is 100 inches tall. This is absurd, but it won't matter because we only need to find the difference between the heights of Janice and Krysta. If Janice is 100 inches tall, then David must be 104 inches tall. If David is 104 inches tall, then Evan is 106 inches tall. If Evan is 106 inches tall, then Kathleen is 93 inches tall. Finally, if Kathleen is 93 inches tall, then Krysta is 99 inches tall, which is $100 - 99 = 1$ inch shorter than Janice. Regardless of what height is assigned to Janice, their height difference is 1 inch.

2019-2020 School Handbook, #165

The odds of success are a ratio of the form $A:B$, where A is the probability of a successful outcome and B is the probability of an unsuccessful outcome. If the odds of winning the game Galaxy Quest are 3:4, what is the probability of losing this game? Express your answer as a decimal to the nearest hundredth.

Since the odds of winning the game are 3 to 4, it follows that the probability of losing is $4/(3 + 4) = 4/7 \approx 0.57$.

2020 Chapter Sprint Round, #21

Noah wants to fill in the two blanks in the numeral 5__1__2 to create a five-digit positive integer that is divisible by 6. What is the greatest five-digit multiple of 6 that he can create?

*An integer is divisible by 6 if and only if it is divisible by 2 and by 3. An integer is divisible by 2 if and only if its units digit is 0, 2, 4, 6 or 8, and the number in question has units digit 2, so it is divisible by 2. An integer is divisible by 3 if and only if the sum of its digits is divisible by 3. The sum of the three known digits is $5 + 1 + 2 = 8$, which is 1 short of being a multiple of 3. Therefore, the two undetermined digits must add to a multiple of 3 plus 1; because digits cannot exceed 9, the sum of the two undetermined digits cannot exceed 18. As a result, the two digits in question can have a sum of 1, 4, 7, 10, 13 or 16. The desire is for the maximum value, which is 16 for this sum. A 9 in the left blank and a 7 in the right blank yields the greatest result, **59,172**.*

2020 Chapter Target Round, #4

The movie sequel *Cat Lawyer II: The Purrfect Crime* grossed \$35.3 million during its opening weekend by selling 4.29 million tickets. Based on this, what was the average price per ticket during opening weekend for this movie?

Dividing, we get $(\$35.3 \times 10^6)/(4.29 \times 10^6) \approx \mathbf{\$8.23}$.

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